Nucleon-Nucleon Interactions from Lattice QCD

Takumi Doi

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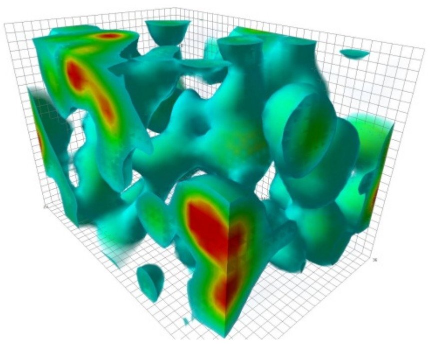
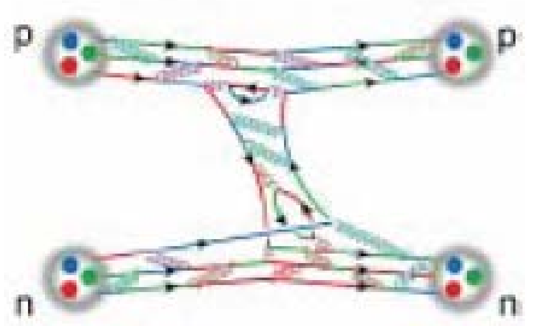
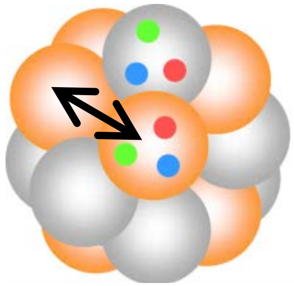






Outline of the Lecture

- Lecture 1
  - – Introduction
  - – Review of lattice QCD simulations (c.f. R.Gavai)
  - – Quick overview of the framework (HAL QCD method)
  - – Review of scattering problems
- Lecture 2 (tutorial)
  - – Nambu-Bethe-Salpeter (NBS) wave function
    - Derivation of it’s asymptotic behavior
  - – Scatterings on the lattice
    - Derivation of Lushcer’s formula

Outline of the Lecture

- Lecture 3
  - Application to Nucleon-Nucleon (NN) interaction
    - Energy independent potential
  - Other two-baryon interactions w/ hyperons

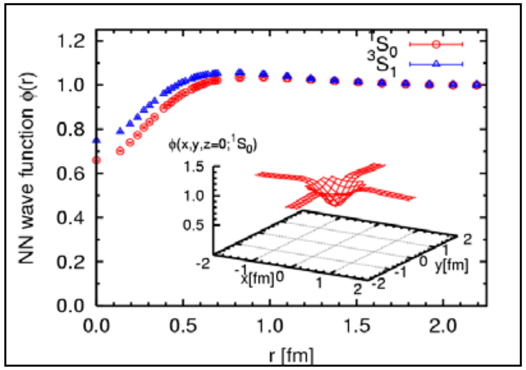
- Lecture 4
  - Application to Three-Nucleon (3N) interaction
    - Unified contraction algorithm
  - Summary / Prospects

Any questions are welcome !

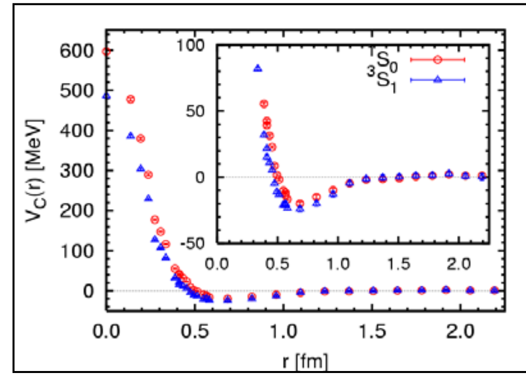
Our Approach [HAL QCD method]

Lattice QCD

NBS wave func.



Lat Nuclear Force



$$\psi_{NBS}(\vec{r}) \;=\; \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq\; e^{i\delta(k)}\sin(kr+\delta(k))/(kr)$$

(at asymptotic region)

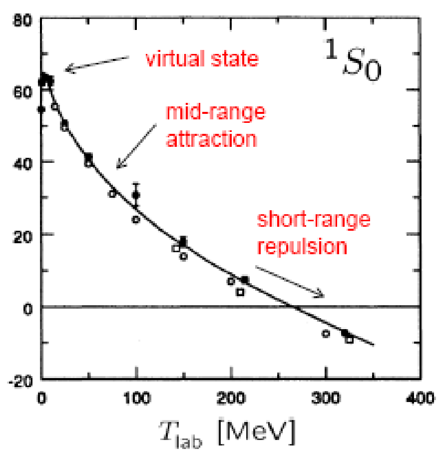
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}\,'\,U(\vec{r},\vec{r}\,')\psi(\vec{r}\,')$$

Lat potential is faithful to phase shift by construction

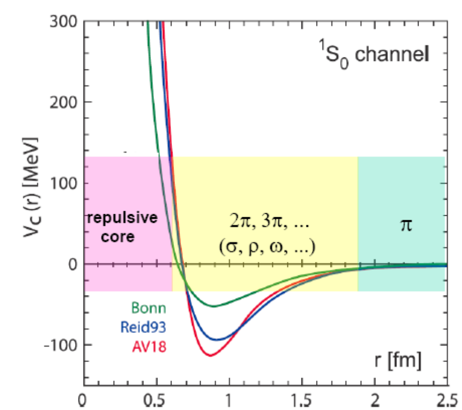
Analog to …

Scattering Exp.

Phase shifts



Phen. Potential



Nuclear Forces from Lattice QCD [HAL QCD method]

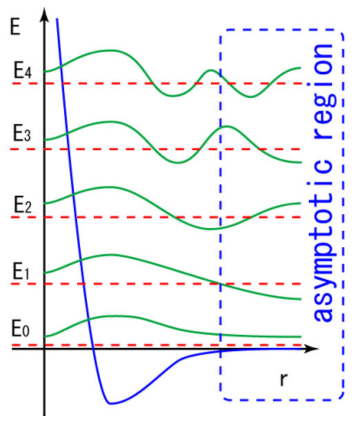
- Potential is constructed so as to reproduce the NN phase shifts (or, S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 2N \rangle$$

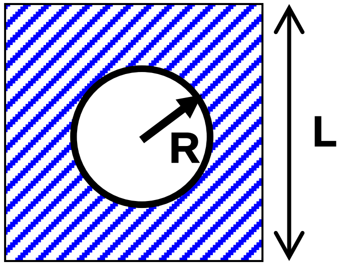
$$E = 2\sqrt{m^2+k^2}$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$



  – Wave function ←→ phase shifts

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

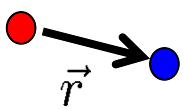


M.Luscher, NPB354(1991)531
CP-PACS Coll., PRD71(2005)094504
C.-J.Lin et al., NPB619(2001)467
Ishizuka, PoS LAT2009 (2009) 119

How to calculate NBS wave function on the lattice ?

- 4pt correlation function

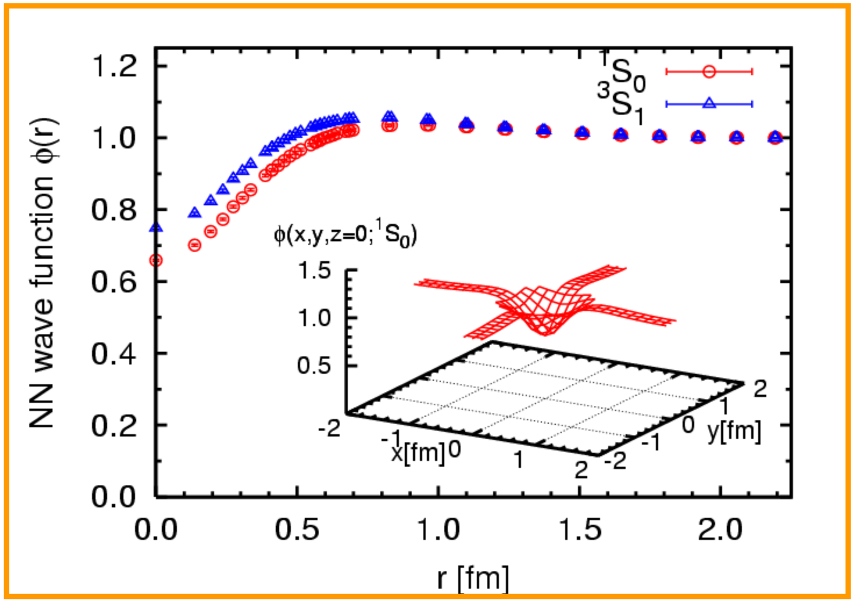
$$
\begin{aligned}
G(\vec{r}, t - t_0) &= \sum_{\vec{x}} \langle 0 | N(\vec{r} + \vec{x}, t) N(\vec{x}, t) \overline{NN}(t_0) | 0 \rangle \\
&= \sum_{n} e^{-E_n (t - t_0)} \sum_{\vec{x}} \langle 0 | N(\vec{r} + \vec{x}, 0) N(\vec{x}, 0) | E_n \rangle \langle E_n | \overline{NN}(0) \rangle \\
&\simeq e^{-E_0 (t - t_0)} \sum_{\vec{x}} \langle 0 | N(\vec{r} + \vec{x}, 0) N(\vec{x}, 0) | E_0 \rangle \langle E_0 | \overline{NN}(0) \rangle
\end{aligned}
$$



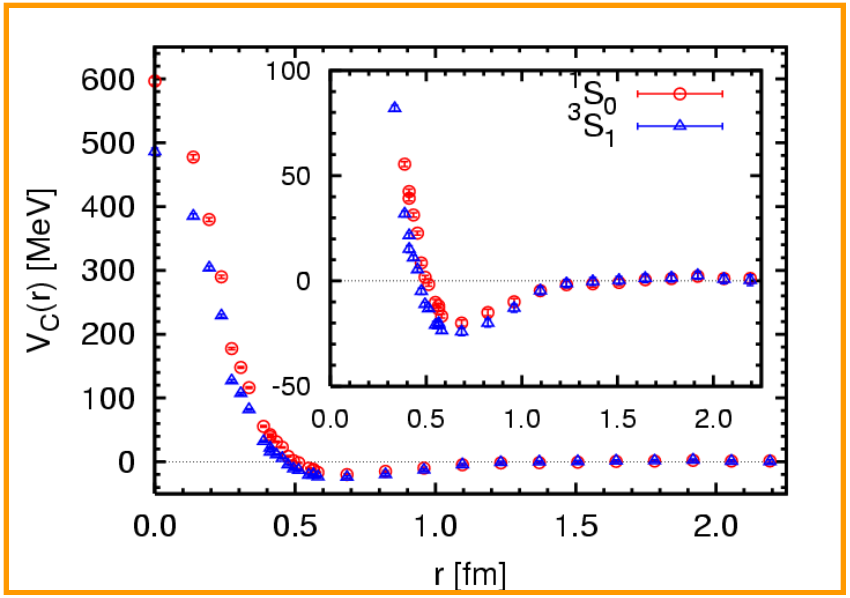
  - Extract the NBS wave function of ground state (g.s.) after the saturation (t >> t0)

Nuclear Potential (from Lat QCD)

NBS wave function



Nuclear Force



<ins>**Quenched**</ins> QCD

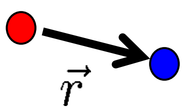
m_π = 530MeV, L=4.4fm

Ishii-Aoki-Hatsuda,
PRL99(2007)022001

How to calculate NBS wave function on the lattice ?

- 4pt correlation function

$$
\begin{aligned}
G(\vec{r}, t - t_0) &= \sum_{\vec{x}} \langle 0 | N(\vec{r} + \vec{x}, t) N(\vec{x}, t) \overline{NN}(t_0) | 0 \rangle \\
&= \sum_{n} e^{-E_n (t - t_0)} \sum_{\vec{x}} \langle 0 | N(\vec{r} + \vec{x}, 0) N(\vec{x}, 0) | E_n \rangle \langle E_n | \overline{NN}(0) \rangle \\
&\simeq e^{-E_0 (t - t_0)} \sum_{\vec{x}} \langle 0 | N(\vec{r} + \vec{x}, 0) N(\vec{x}, 0) | E_0 \rangle \langle E_0 | \overline{NN}(0) \rangle
\end{aligned}
$$



  - Extract the NBS wave function of ground state (g.s.) after the saturation (t >> t0)
  - Toward more quantitative results:
    - **G.S. saturation is really feasible to achieve ?**

The Challenge

- S/N issue at light mass

Parisi, Lepage (1989)

  - To achieve ground state saturation, take $t \to \infty$

Single nucleon

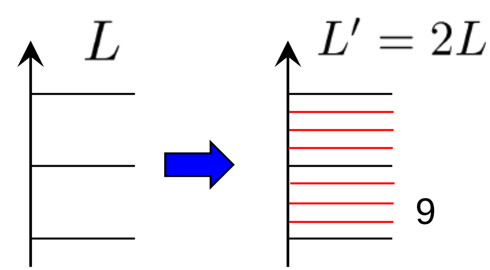
$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\langle N(t)\bar{N}(0)\rangle}{\sqrt{\langle N\bar{N}(t)N\bar{N}(0)\rangle}} \sim \frac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \mathbf{exp}[-(\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Nucleons w/ mass number = A

$$\frac{\text{Signal}}{\text{Noise}} \sim \mathbf{exp}[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

  - Situation gets worse for larger volume
    🡐 large spectral density by scatt. states

$$\Delta E \simeq \frac{\vec{p}^2}{m_N} \simeq \frac{1}{m_N}\left(\frac{2\pi}{L}\right)^2 \simeq 15\text{MeV} \quad \text{for } L = 10\text{fm}$$



➔ Very large t >~ 100 would be required !

Solution

- Central feature:
  - Energy-independence of the potential
  - Existence proof is possible

$$U(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{m}\sum_{n,n'}^{n_{\rm th}}(\nabla^2_{\boldsymbol{r}} + k_n^2)\psi_n(\boldsymbol{r})\mathcal{N}^{-1}_{nn'}\psi^*_{n'}(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r}\,\psi^*_n(\boldsymbol{r})\psi_{n'}(\boldsymbol{r})$$

  - Non-locality of the pot. ➔ derivative expansion

Okubo-Marshak(1958)

$$U(\vec{r},\vec{r'}) = \underbrace{V_c(r)}_{\text{LO}} + \underbrace{S_{12}V_T(r)}_{\text{LO}} + \underbrace{\vec{L}\cdot\vec{S}V_{LS}(r)}_{\text{NLO}} + \underbrace{\mathcal{O}(\nabla^2)}_{\text{NNLO}}$$

Aoki-Hatsuda-Ishii PTP123(2010)89

Aoki et al. arXiv:1212.4896 [hap-lat]

Check on convergence: K.Murano et al., PTP125(2011)1225

Most general form of the potential

$$V(\vec{r}_1, \vec{r}_2, \vec{\nabla}_1, \vec{\nabla}_2; \vec{\sigma}_1, \vec{\sigma}_2)$$

Okubo-Marshak(1958)

- Imposed condition
  - Hermiticity $\quad V^\dagger = V$
  - Energy/Momentum conservation $\quad V(\vec{r}, \vec{\nabla}_1, \vec{\nabla}_2; \vec{\sigma}_1, \vec{\sigma}_2), \;\; \vec{r} = \vec{r}_1 - \vec{r}_2$
  - Galilei invariance $\quad V(\vec{r}, \vec{\nabla}_r; \vec{\sigma}_1, \vec{\sigma}_2)$
  - Rotational invariance $\quad V: \text{ scalar}$
  - Parity conservation $\quad V(\vec{r}, \vec{\nabla}_r; \vec{\sigma}_1, \vec{\sigma}_2) = V(-\vec{r}, -\vec{\nabla}_r; \vec{\sigma}_1, \vec{\sigma}_2)$
  - Time reversal $\quad V(\vec{r}, \vec{\nabla}_r; \vec{\sigma}_1, \vec{\sigma}_2) = V(\vec{r}, -\vec{\nabla}_r; -\vec{\sigma}_1, -\vec{\sigma}_2)$
  - Pauli principle $\quad V(\vec{r}, \vec{\nabla}_r; \vec{\sigma}_1, \vec{\sigma}_2) = V(-\vec{r}, -\vec{\nabla}_r; \vec{\sigma}_2, \vec{\sigma}_1)$
- LO

$$1 \text{ (unit operator)}, \;\; (\vec{\sigma}_1 \cdot \vec{\sigma}_2), \;\; S_{12} \equiv 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

- NLO

$$(\vec{L} \cdot \vec{S})$$

Independent DoF in Isospin space:

$$1 \text{ (unit op.)}, \quad (\vec{\tau}_1 \cdot \vec{\tau}_2)$$

Solution: Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

They are ***not contaminations***, ***but signals***

➔ **Schrodinger Eq. : time-independent ➔ time-dependent**

$$\left(-\frac{\partial}{\partial t}+\frac{1}{4m}\frac{\partial^2}{\partial t^2}-H_0\right)R(\boldsymbol{r},t)=\int d\boldsymbol{r}'U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t)$$

$$2\sqrt{m^2+k_n^2}=E_n=-\frac{\partial}{\partial_t}$$

Grand State (G.S.) saturation is NOT necessary !

Significant advantage of potential method:

$\Delta E \simeq E_{\rm th}-E\simeq m_\pi \simeq 140\text{MeV}$ ➔ Moderate t >~ 10 would be fine

Explicit Lat calc for I=2 pipi phase shift

Beautiful agreement between

(1) Luscher's formula w/ g.s. saturation

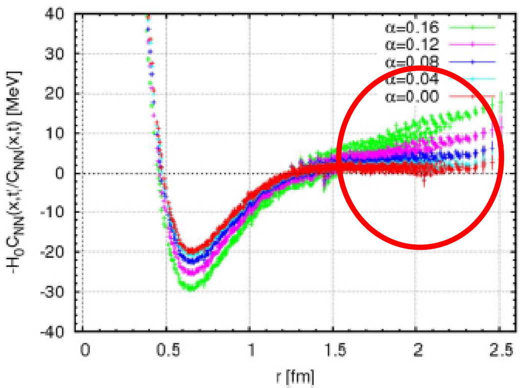
(2) the HAL QCD method w/ & w/o g.s. saturation

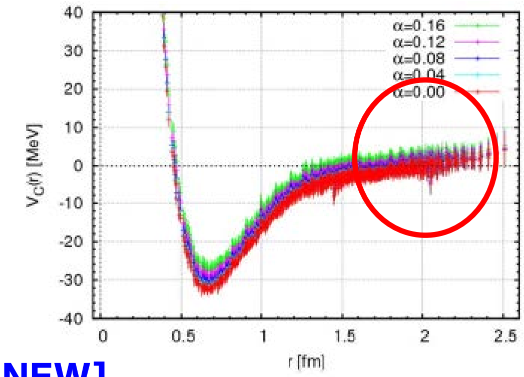
T.Kurth et al. (HAL-BMW Coll.) @ Lat2012

Explicit check on the new t-dep HAL method

NN system

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437





[OLD]

Different sources (creation op.) ➔ different results

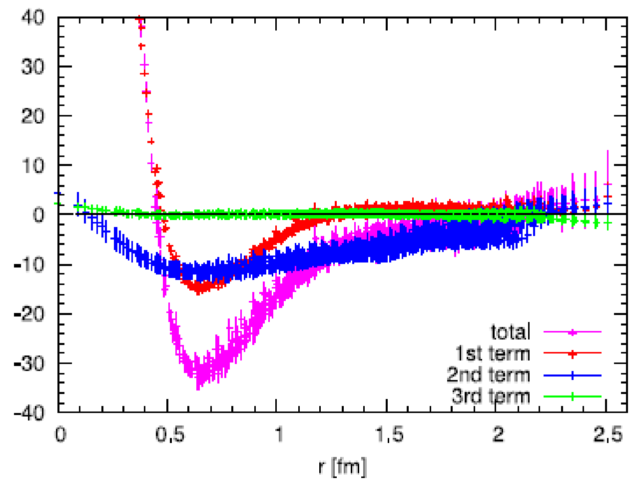
"contaminations" from excited states

[NEW]

Results converged

"signals" from excited states

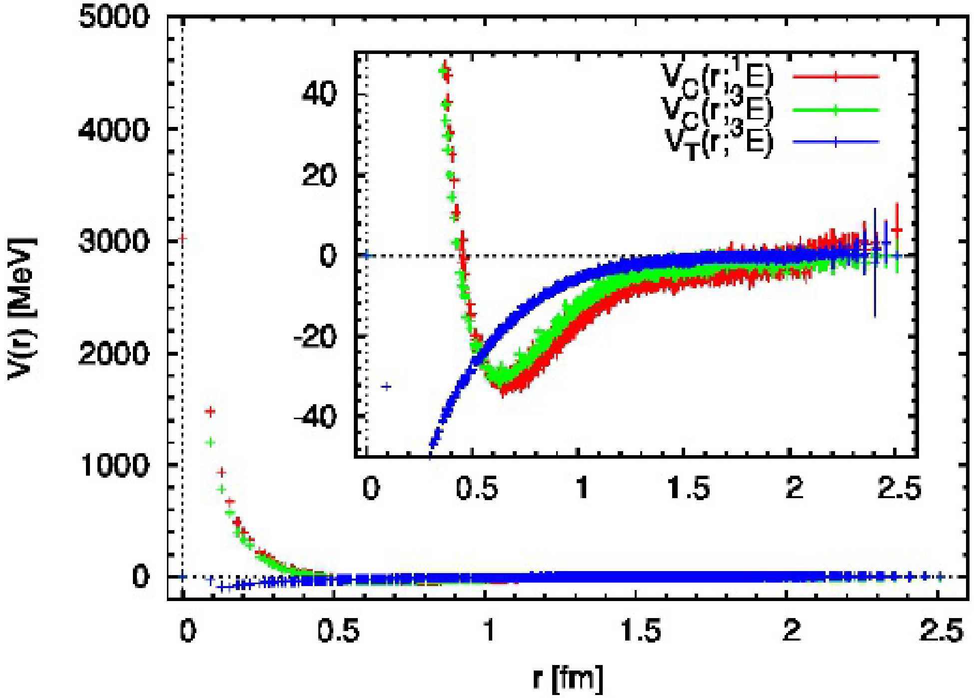
Contribution of each component

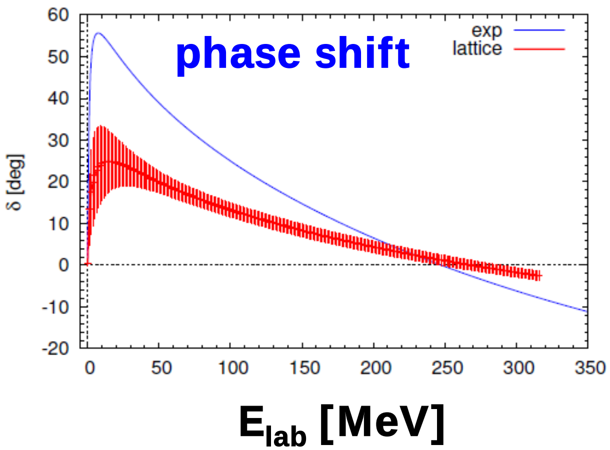


(1) NN potential on the lattice (positive parity)

$^{2S+1}L_J$

- "di-neutron" channel $^1S_0$ ➔ central force
- "deuteron" channel $^3S_1-^3D_1$ ➔ central & tensor force





Not Bound $a(^1S_0) = 1.6(1.1)$ fm

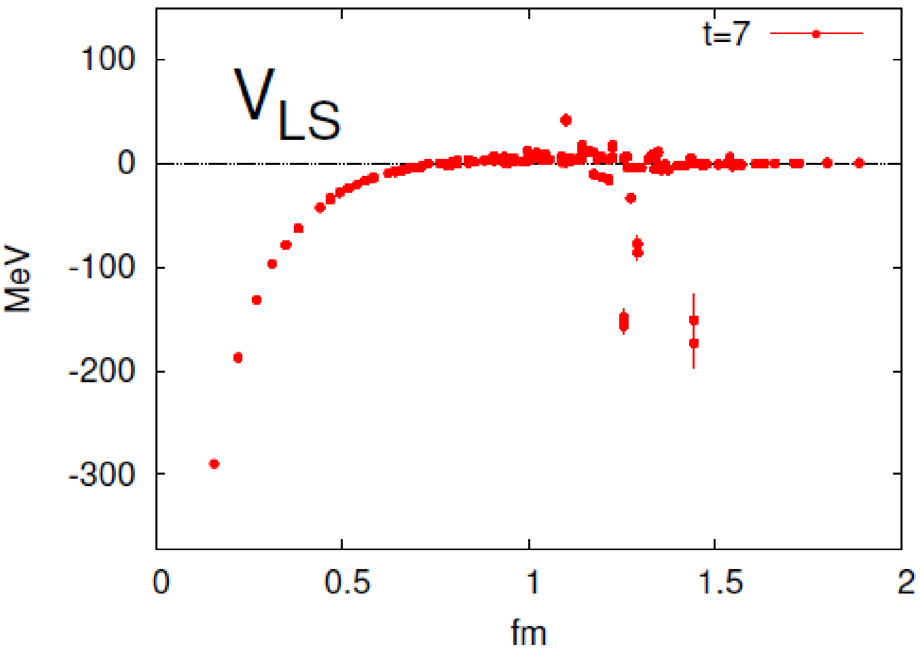
[N.Ishii]

Nf=2+1 clover (PACS-CS), 1/a=2.2GeV,
L=2.9fm, m_π=0.7GeV, m_N=1.6GeV

NN potential on the lattice
(negative parity)

$^{2S+1}L_J$

- S=1 channel: $^3P_0,\ ^3P_1,\ ^3P_2\!-\!^3F_2$
  - Central & tensor forces in LO
  - Spin-orbit force in NLO
    - Inject a momentum ➔ $J^P = A_1^-, T_1^-, T_2^-$



Nf=2 clover (CP-PACS), L=2.5fm, mπ=1.1GeV

$\vec{L}\cdot\vec{S} = +1 \text{ for } ^3P_2$

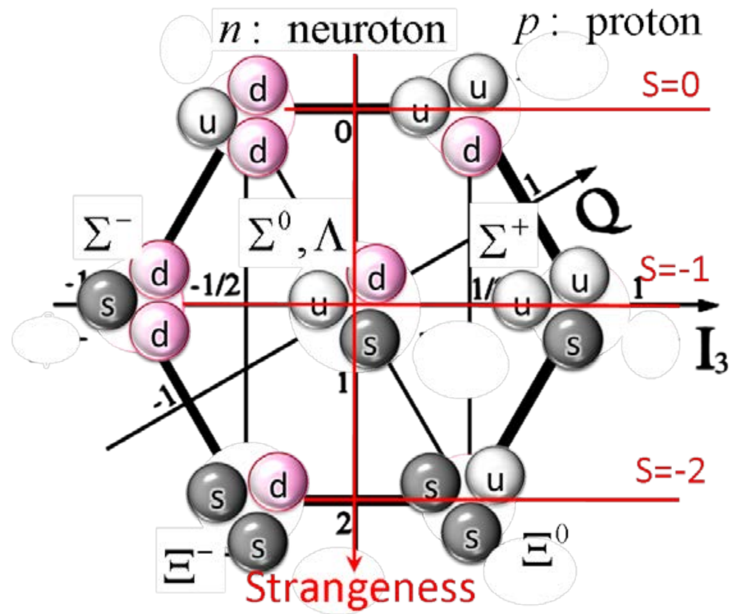
Superfluidity $^3P_2$ in neutron star
←→ neutrino cooling

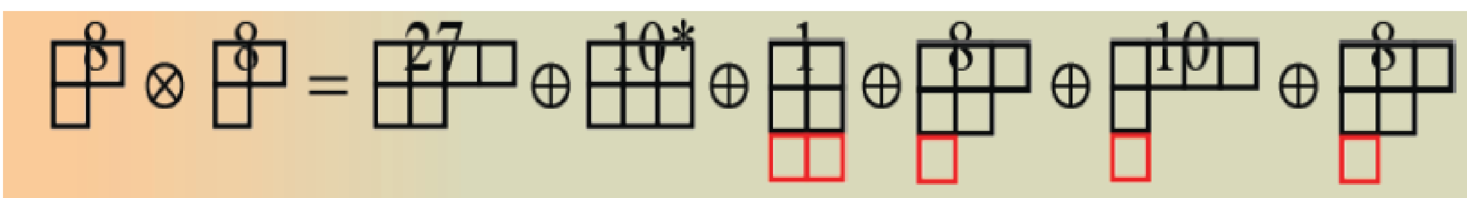
←→ **Cas A NS: cooling is being measured !**

[K.Murano]

Hyperon Forces

New DoF in nuclear physics





8 X 8 = 27 + 8s + 1 + 10* + 10 + 8a

symmetric

anti-symmetric

NN channel

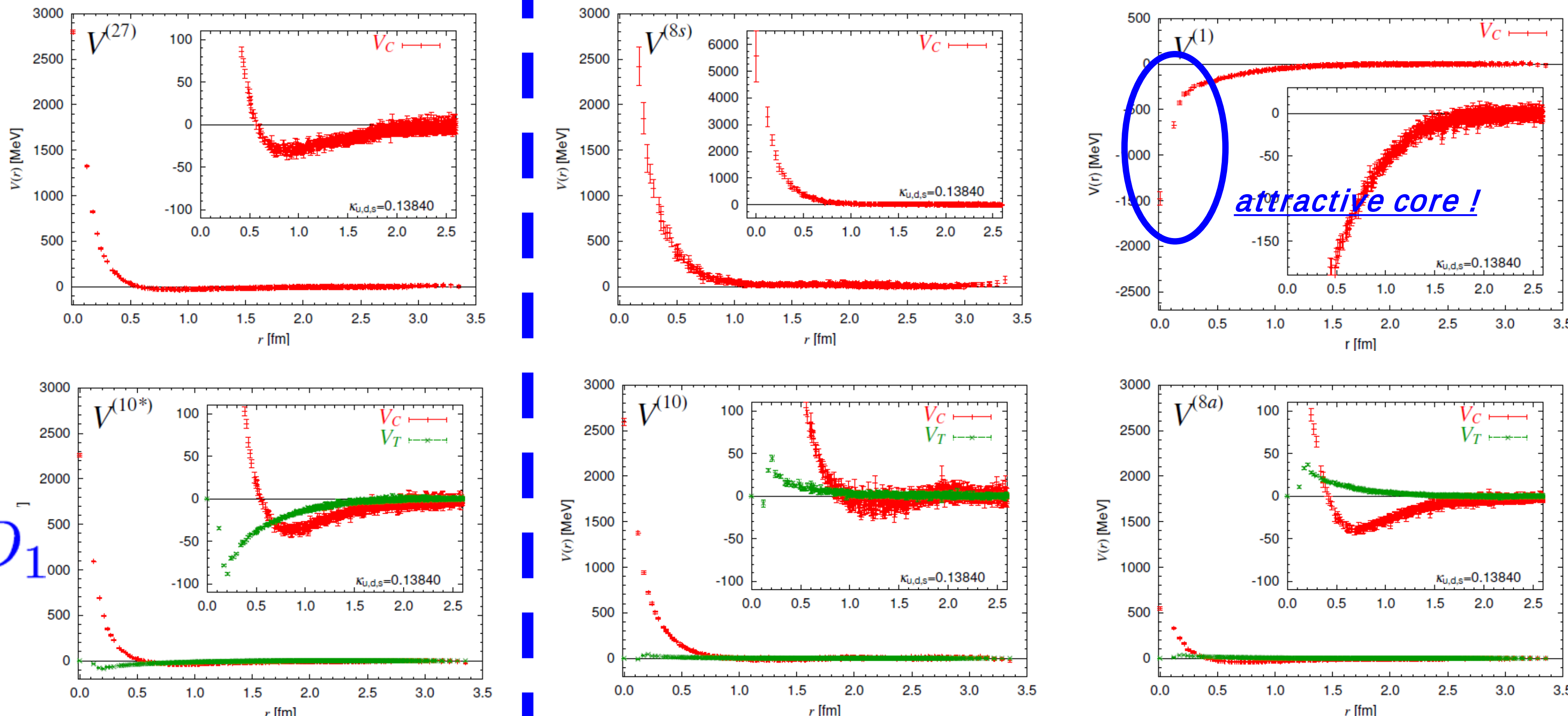
SU(3) study

(2) Hyperon forces

a=0.12fm, L=3.9fm,
m(PS)= 0.47-1.2GeV

$^1S_0$

$^3S_1-^3D_1$



27,10*:
Same as NN

8s,10:
strong repulsive core

1s: deep attractive pocket
8a: weak repulsive core

T.Inoue et al. (HAL QCD Coll.), NPA881(2012)28

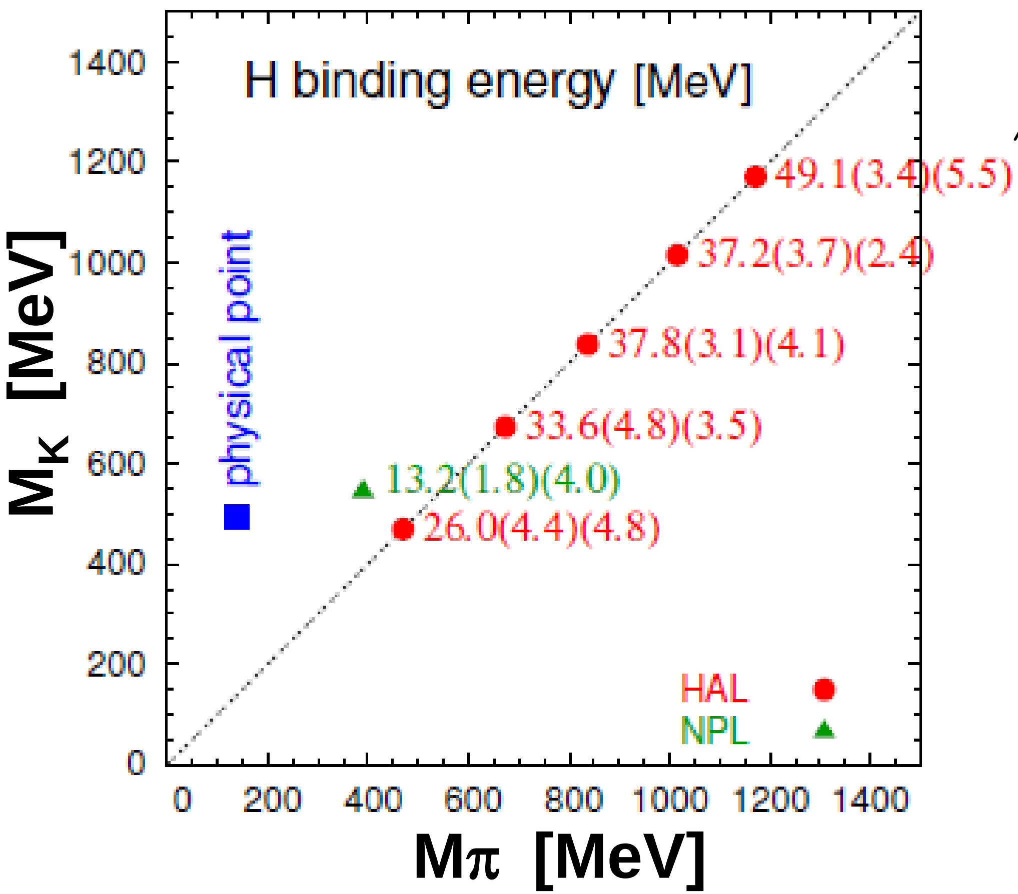
Also seen in Takahashi et al. (2010), Kawanai et al. (2010)
Meson-baryon, **Y.Ikeda** et al., arXiv:1111.2663

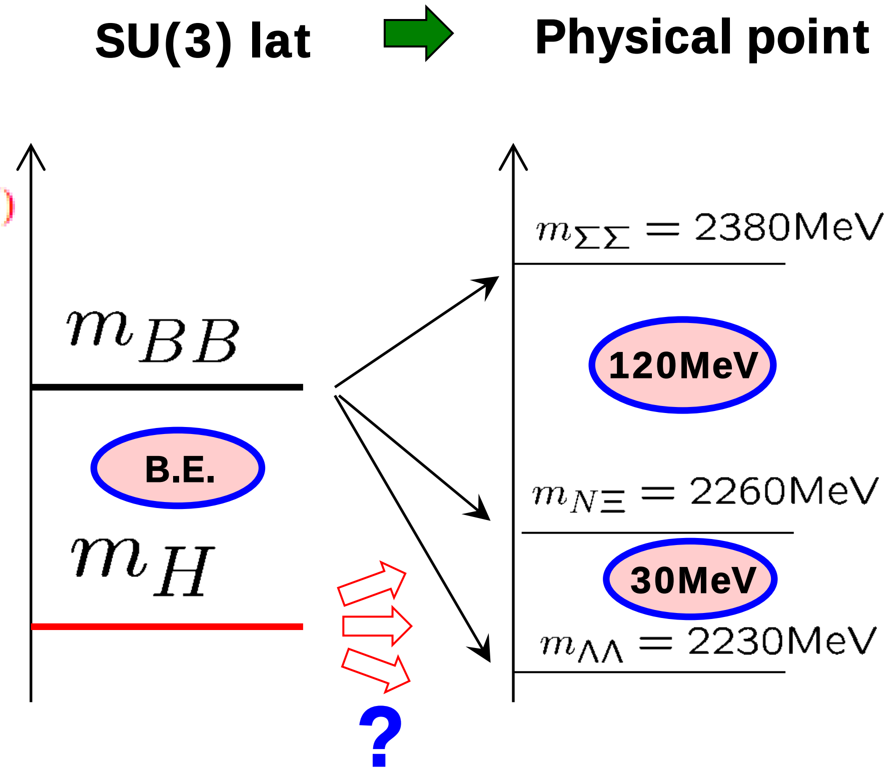
Repulsive core
← Pauli principle !

M.Oka et al., NPA464(1987)700

→ Study of baryonic matter & Neutron Star [T.Inoue]

H-dibaryon (uuddss, I=0, $^1S_0$)



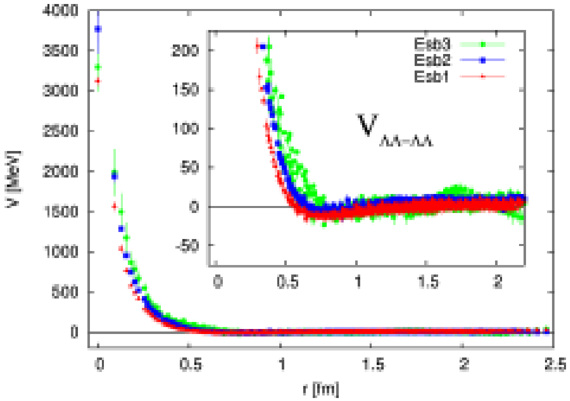


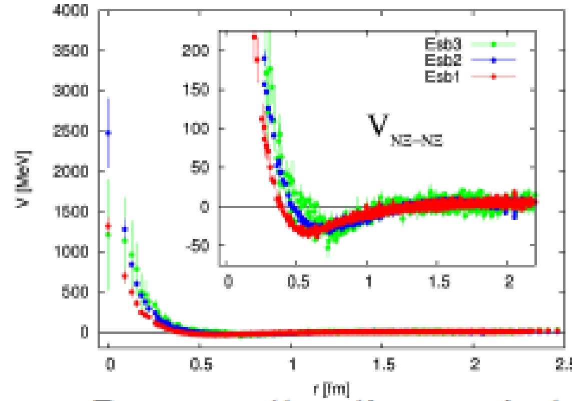
Coupled channel study is essential

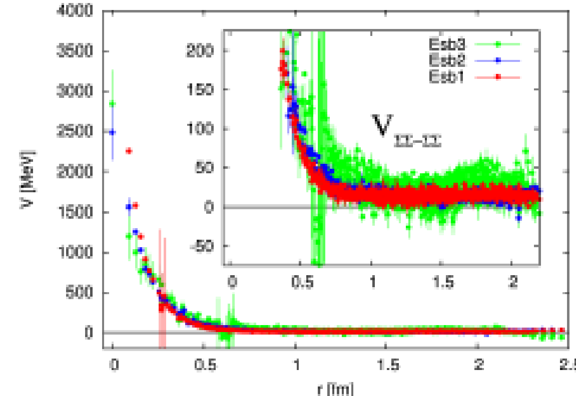
$\Lambda\Lambda\text{–}N\Xi\text{–}\Sigma\Sigma$

➔ K.Sasaki

Coupled channel beyond SU(3)





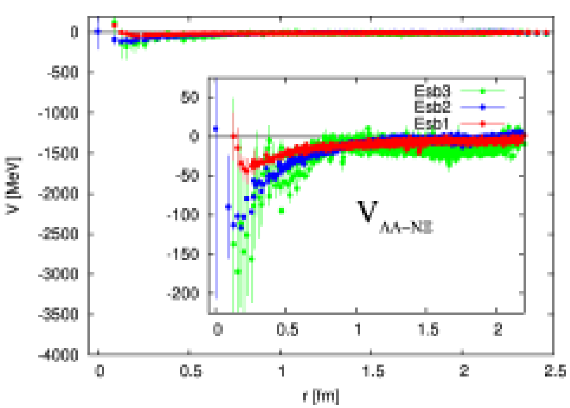


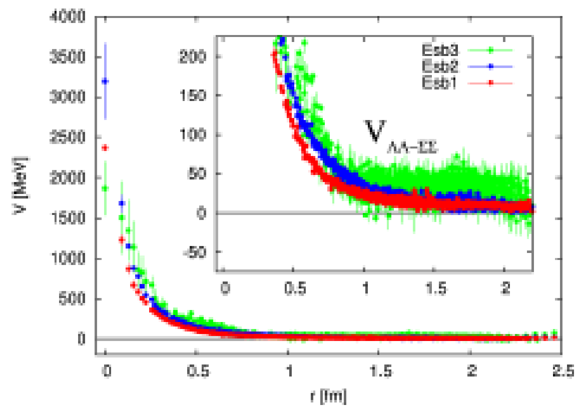
shallow attractive pocket

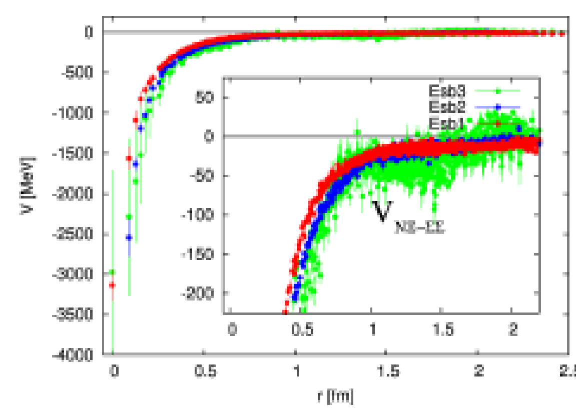
Deeper attractive pocket

Strongly repulsive

All channels have repulsive core







Relatively weaker than the others

[K.Sasaki]

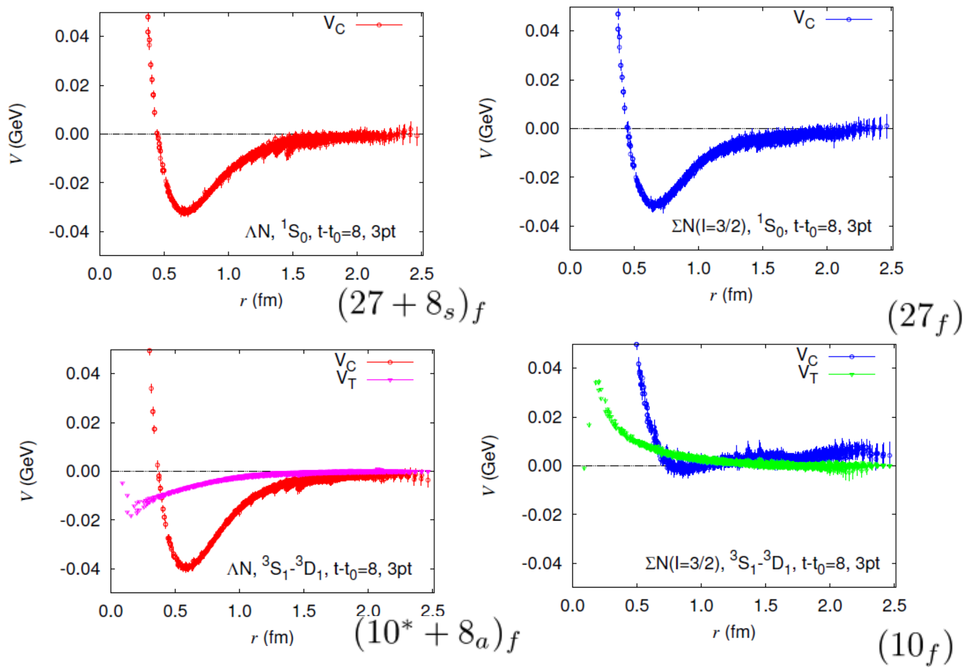
Hyperon Interactions in S= -1

ΛN $\Sigma N(I=3/2)$

(single channel study)

$^1S_0$

$^3S_1$–$^3D_1$



$\Lambda N(^1S_0, ^3S_1)$

$\Sigma N(I=3/2, ^1S_0)$

➔Attractive

Not bound

$\Sigma N(I=3/2, ^3S_1)$

➔Repulsive

Nemura et al.
Nf=2+1, L=2.9fm,
mπ= 0.70GeV
arXiv:1203.3320

Crucial input for the core of neutron star and hyper-nuclei

[H. Nemura]

ΛN-ΣN coupled channel study is also in progress

- Many other applications
  - Meson-meson, incl. resonance channels
  - Meson-baryon system
  - Baryon-baryon incl. decuplet
  - Inter-quark potential
  - and much more…

  - Tomorrow: **Three-body systems**

  - **Very new field (less than 10yrs)**
    YOUR new idea Welcome !

Backup Slides

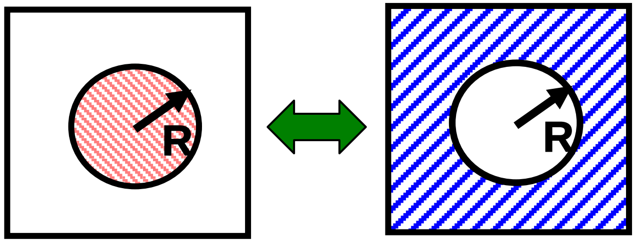
A few remarks on the Lattice Potential

- Potential is NOT an observable and is not unique:
  They are, however, phase-shift equivalent potentials.
  - Choosing the pot. (sink op.) ⇐⇒ choosing the "scheme"

- We study potential (+ phase shifts), since:
  - Convenient to understand physics
  - Essential to study many-body

$$Lat \to \boxed{\delta_E \to U(\boldsymbol{r})} \to \text{many-body}$$

$$\boxed{Lat \to \overset{\to}{\delta_E} \overset{\to}{\leftarrow} U(\boldsymbol{r})} \to \text{many-body}$$



  - Finite V artifact better under control
  - Excited states better under control

Effective Schrodinger equation with E-independent potential

$$K(\vec{x};E) \equiv \left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E)$$

[START] local but E-dep pot. ($L^3 x L^3$ dof)

(1) We assume $\psi(x; E)$ for different E is linearly independent with each other.

(2) $\psi(x; E)$ has a **"left inverse"** as an integration operator as

$$E \equiv 2\sqrt{m_N^2 + k^2}$$

$$\int d^3x \tilde{\psi}(\vec{x}; E')\psi(\vec{x}; E) = 2\pi\delta(E - E')$$

(3) K(x; E) can be factorized as

$$\begin{aligned} K(\vec{x}; E) &= \int \frac{dE'}{2\pi} K(\vec{x}; E') \times \int d^3y \tilde{\psi}(\vec{y}; E')\psi(\vec{y}; E) \\ &= \int d^3y \left\{ \sum_\alpha \int \frac{dE'}{2\pi} K(\vec{x}; E')\tilde{\psi}(\vec{y}; E') \right\} \psi(\vec{y}; E) \end{aligned}$$

$$\equiv m_N U(\vec{x}, \vec{y})$$

(4) We are left with **an effective Schrodinger equation with an E−independent potential U.**

$$\left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E) = m_N \int d^3y\, U(\vec{x},\vec{y})\psi(\vec{y};E)$$

Intuitive understanding

[GOAL] non-local but E-indep pot. ($L^3 x L^3$ dof)

Asymptotic form of BS wave function

[C.-J.D.Lin et al., NPB619,467(2001)]

For simplicity, we consider BS wave function of two pions

$$\psi_{\vec{q}}(\vec{x}) \equiv \left\langle 0 \middle| N(\vec{x})\, N(\vec{0}) \middle| N(\vec{q}) N(-\vec{q}), in \right\rangle$$

complete set

$$1 = \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec{p})} |N(\vec{p})\rangle\langle N(\vec{p})| + \cdots$$

$$= \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec{p})} \langle 0 | N(\vec{x}) | N(\vec{p}) \rangle \left\langle N(\vec{p}) \middle| N(\vec{0}) \middle| N(\vec{q}) N(-\vec{q}), in \right\rangle + I(\vec{x})$$

$$Z^{1/2} e^{i\vec{p}\cdot\vec{x}}$$

$$disc. + Z^{1/2} \frac{T(\vec{p};\vec{q})}{m_N^2 - (2E_N(\vec{q}) - E_N(\vec{p}))^2 + \vec{p}^2 - i\varepsilon}$$

$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2E_N(\vec{p})} \frac{T(\vec{p};\vec{q})}{4E_N(\vec{q}) \cdot (E_N(\vec{p}) - E_N(\vec{q}) - i\varepsilon)} e^{i\vec{p}\cdot\vec{x}} \right)$$

Integral is dominated by the on-shell contribution $E_N(\vec{p}) \approx E_N(\vec{q})$

➔ T-matrix becomes the on-shell T-matrix

$$T^{(s-wave)}(s) = \frac{E(\vec{q})}{2|\vec{q}|}(-i)\left(e^{2i\delta_0(s)} - 1\right)$$

$$= Z\left(e^{i\vec{q}\cdot\vec{x}} + \frac{1}{2i}\left(e^{2i\delta_0(s)} - 1\right)\frac{e^{iqr}}{qr} \right) + \cdots$$

The asymptotic form

$$\psi_{\vec{q}}(\vec{x}) = Z e^{i\delta_0(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \quad \text{(s-wave)}$$

This is analogous to a non-rela. wave function