Nucleon-Nucleon Interactions from Lattice QCD

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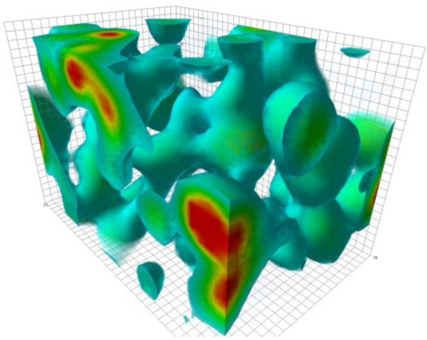
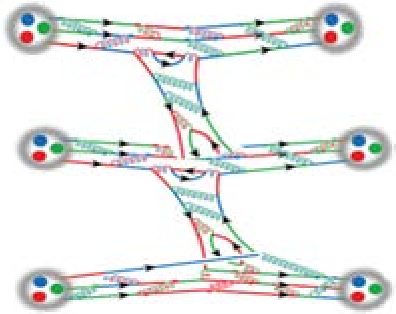
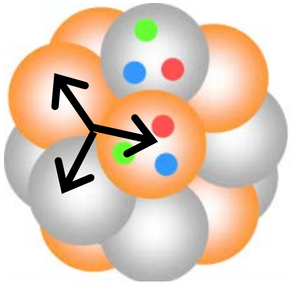






Outline of the Lecture

- Lecture 1
  - Introduction
  - Review of lattice QCD simulations (c.f. R.Gavai)
  - Quick overview of the framework (HAL QCD method)
  - Review of scattering problems

- Lecture 2 (tutorial)
  - Nambu-Bethe-Salpeter (NBS) wave function
    - Derivation of it's asymptotic behavior
  - Scatterings on the lattice
    - Derivation of Lushcer's formula

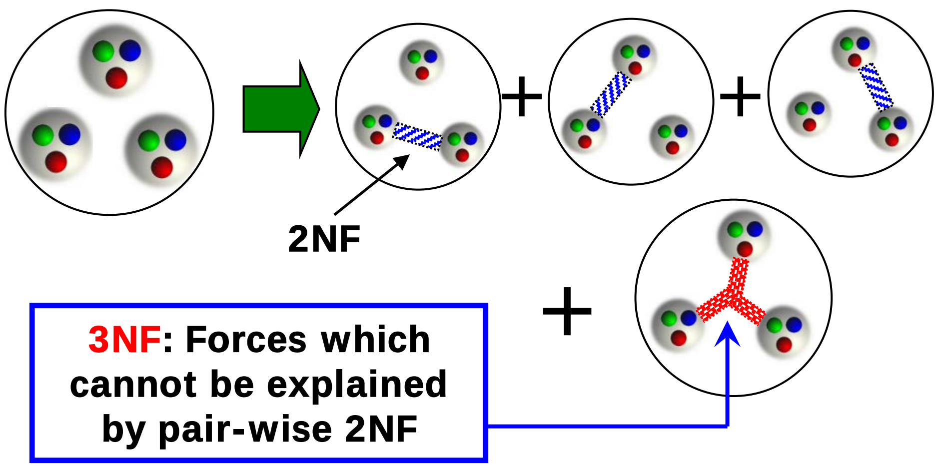
Outline of the Lecture

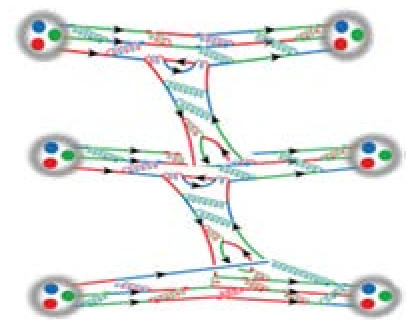
- Lecture 3
  - Application to Nucleon-Nucleon (NN) interaction
    - Energy independent potential
  - Other two-baryon interactions w/ hyperons
- Lecture 4
  - Application to Three-Nucleon (3N) interaction
    - Unified contraction algorithm
  - Summary / Prospects

Any questions are welcome !

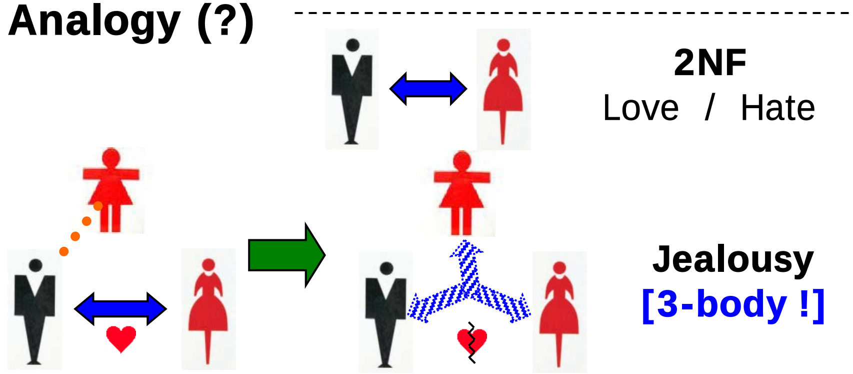
<u>Frontier</u> in Hadron-Hadron Interactions
⇒Three-Nucleon Forces (3NF)

What is 3NF ?



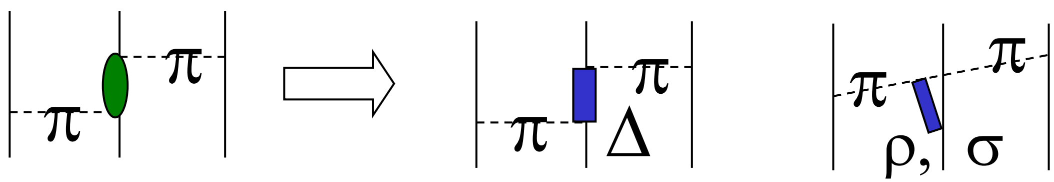


Analogy (?)

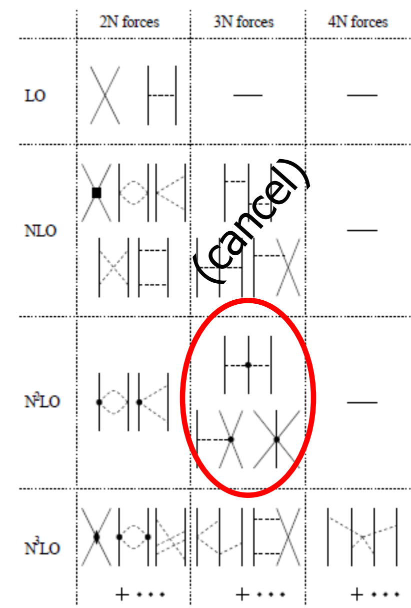


Three-Nucleon Forces (3NF)

- It is natural to expect the existence of 3NF
- It is very nontrivial to determine 3NF from QCD
- 2πE-3NF Fujita-Miyazawa, PTP17(1957)360
  - Off-energy-shell πN scatt



- Phenomenological models
  - Fujita-Miyazawa, Tucson-Melbourne, Urbana/Illinois, …
- EFT expansion ➔ 3NF appear at NNLO



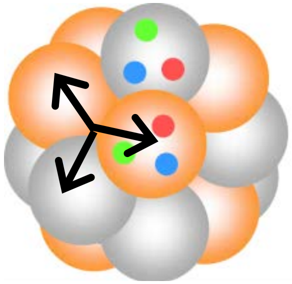
- Are 3NF really important in physics ?

U.v.Kolck, PRC49(1994)2932

Epelbaum, Prog.Part.Nucl.Phys.57(06)654

Frontier in Hadron-Hadron Interactions
⇒**Three-Nucleon Forces (3NF)**

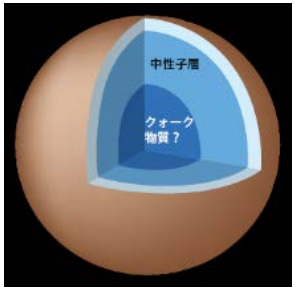
◆ *B.E. of light nuclei*



◆ *Neutron rich nuclei*

➔ *Nucleosynthesis*

◆ *Nucleon-deuteron scattering*



◆ ***EoS** of nuclear matter*

➔ *Saturation point*

➔ *Neutron Star / SuperNova*

3NF in Few-Body Systems

- Precise few-body calc:
  - e.g. benchmark calc of ^{4}He by 7 methods (2NF only)

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

➔ **0.5 % prec. for B.E.**

H.Kamada et al., PRC64(2001)044001

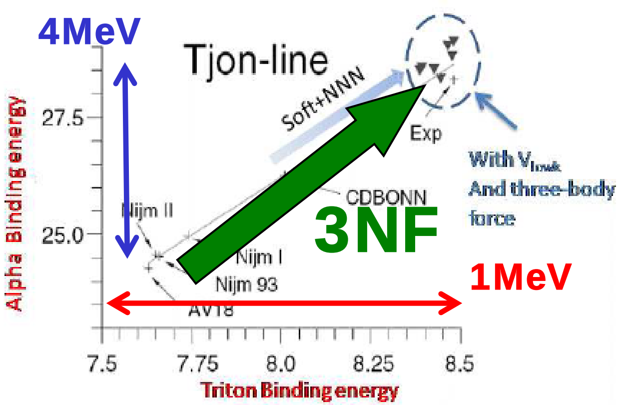
- 2N force cannot reproduce B.E.

δB.E.= 0.5-1MeV for ^{3}H
δB.E.= 2-4 MeV for ^{4}He

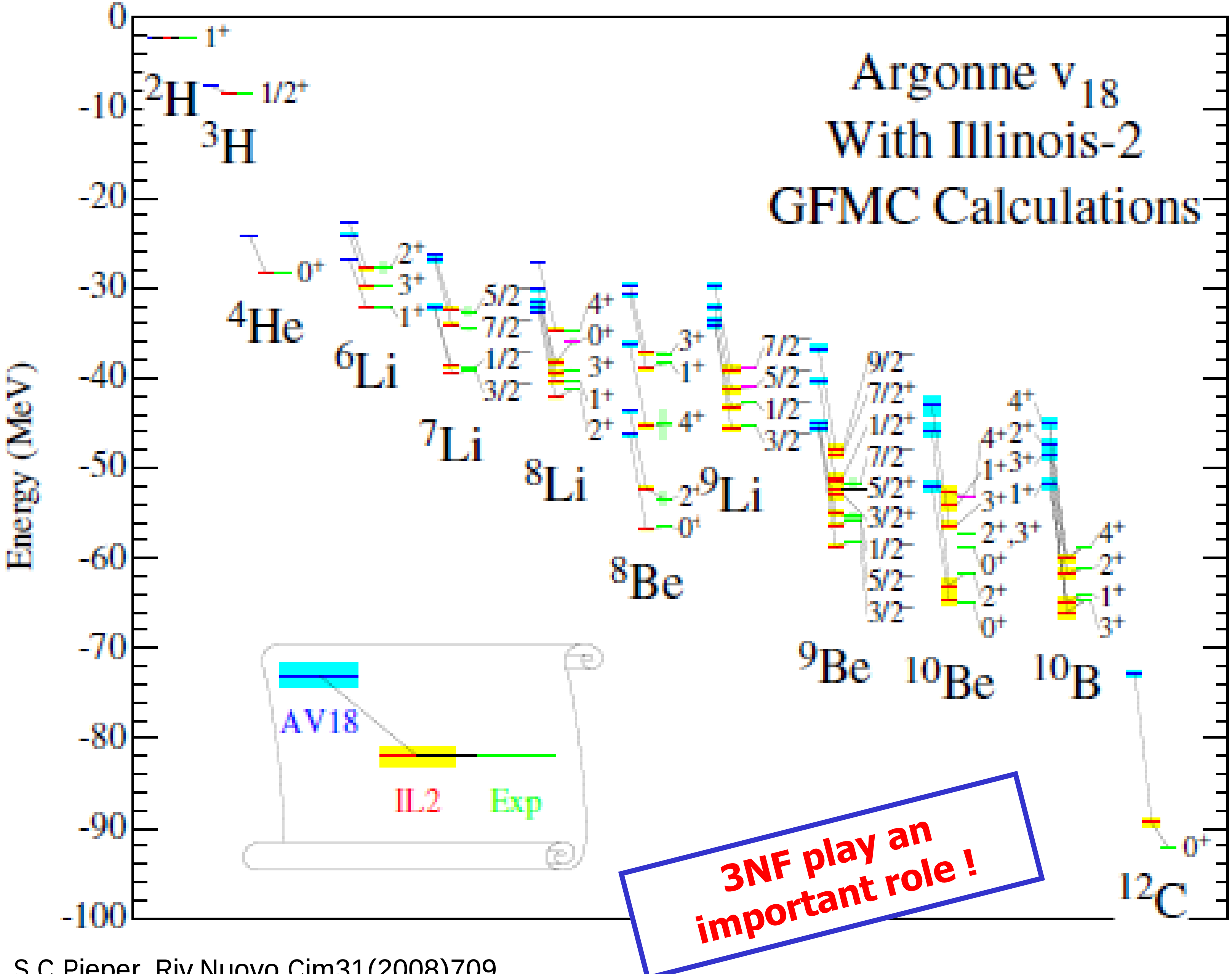
missing

c.f. δB.E.= 0.35MeV for ^{3}H in fss2

➔ Attractive 3NF necessary



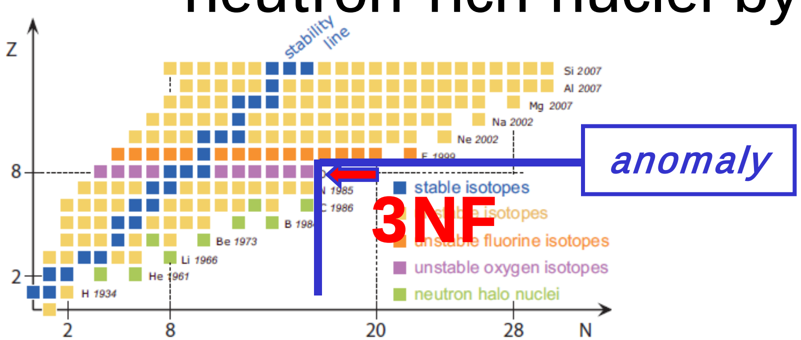
Nogga et al., PRL85(2000)944



S.C.Pieper, Riv.Nuovo.Cim31(2008)709
arXiv:0711.1500

3NF in Neutron-Rich Nuclei

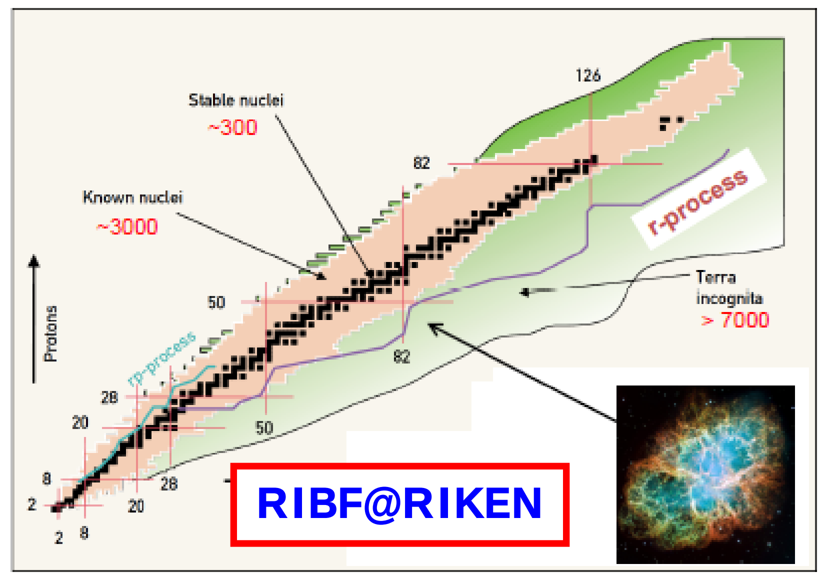
- The effect on the nuclear chart
  - Anomaly in drip line and nontrivial magic number in neutron-rich nuclei by 3NF



drip line: $^{28}O \rightarrow {}^{24}O$

nontrivial magic number
N=28 for $_{20}Ca$

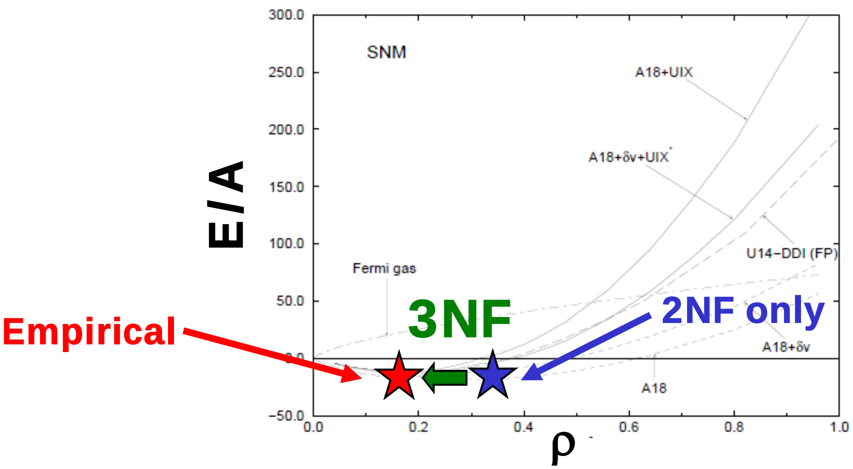
T.Otsuka et al., PRL105(2010)032501
J.D.Holt et al., arXiv:1009.5984



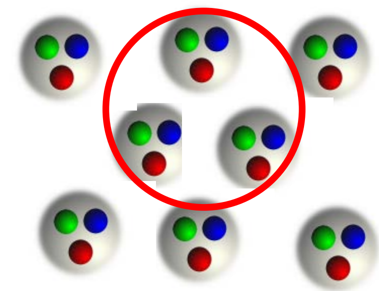
Nucleosynthesis by Supernova

3NF in symmetric nuclear matter

- Saturation point of nuclear matter requires 3NF



Repulsive 3NF
also necessary



A.Akmal et al., PRC58(1998)1804

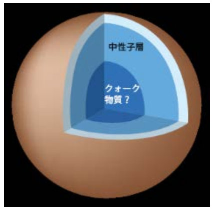
$$\rho_0 = 0.16 \text{ fm}^{-3}$$

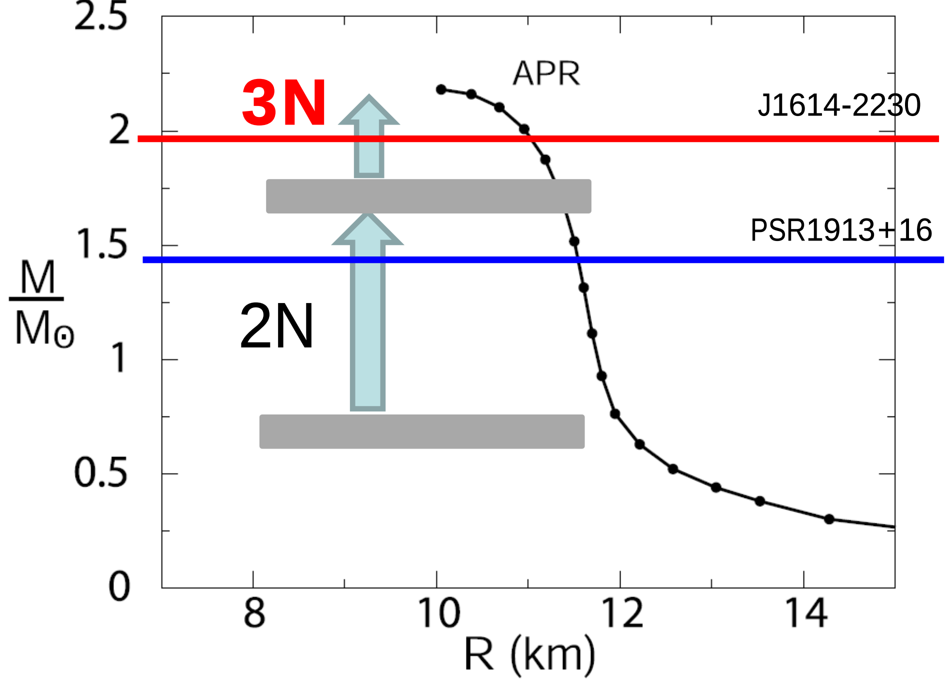
$$E/A = 16 \text{ MeV}$$

Crucial role of 3NF in nuclear matter

Neutron Star

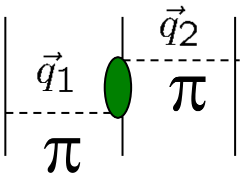
(Densest system in the Universe)





Short-range repulsive 3NF is required

In phenomenological models:



2πE-3NF:
➔ too strong attraction

Tucson-Melbourne

$$\sim \frac{F_\pi^2(\vec{q}_1^{\,2})(\vec{\sigma}\cdot\vec{q}_1)}{\vec{q}_1^{\,2}+m_\pi^2} F(\vec{q}_1,\vec{q}_2) \frac{F_\pi^2(\vec{q}_2^{\,2})(\vec{\sigma}\cdot\vec{q}_2)}{\vec{q}_2^{\,2}+m_\pi^2}$$

$$F_\pi(\vec{q}^{\,2}) = \left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2+\vec{q}^{\,2}}\right) \qquad \Lambda_\pi \sim 700\text{MeV} \ll 1.3\text{GeV}$$

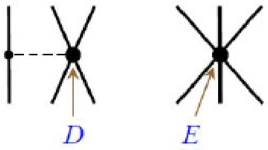
➔ (effective) short-range repulsion by cut-off

Urbana/Illinois

2πE-3NF + explicit short-range repulsive term

Chiral EFT

Short-range LECs fitted



Can we understand it directly from QCD ?

EoS of Neutron Stars through Gravitational Waves ?

- Full GR simulation of binary neutron star mergers w/ Shen-EoS (stiff) and Hyperon(Λ)–EoS (soft)

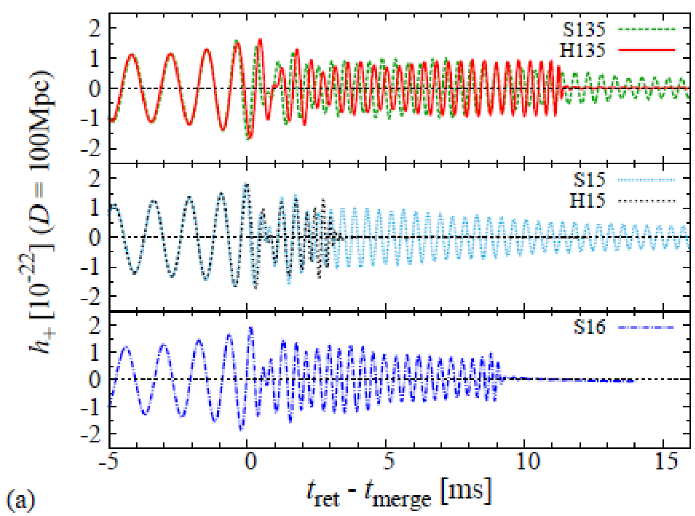


FIG. 4: (a) GWs observed along the axis perpendicular to t
D = 100 Mpc. (b) The effective amplitude of GWs defined by
noise amplitudes of a broadband configuration of Advanced Las
and Large-scale Cryogenic Gravitational wave Telescope (LCGT

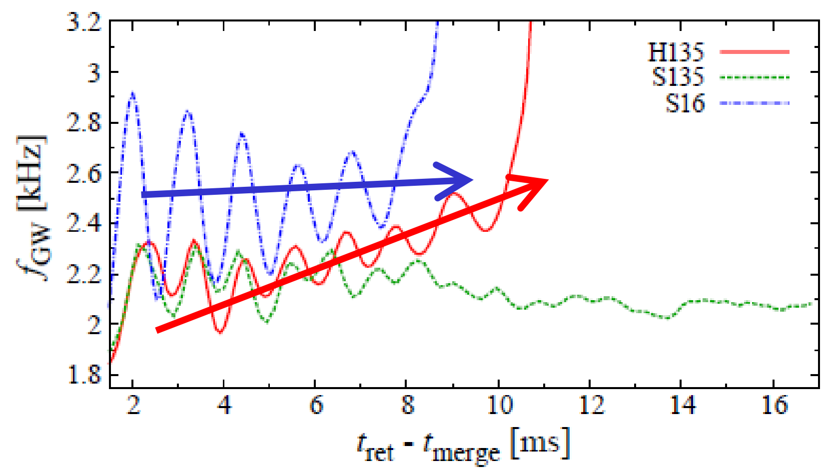
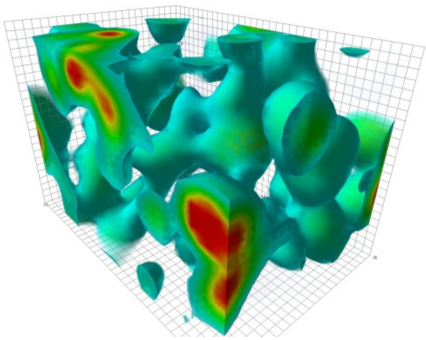


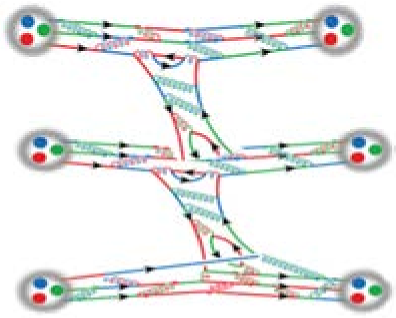
FIG. 5: $f_{GW}(t)$ in the HMNS phase, smoothed by a weighted spline, for H135 (solid red), S135 (dashed green), and S16 (dashed-dotted blue).

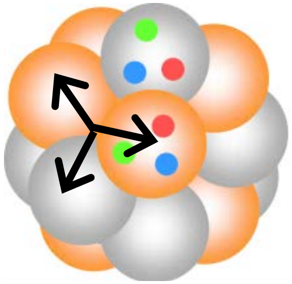
Y.Sekiguchi et al., arXiv:1110.4442[astro-ph.HE]

Three Nuclear forces

from Lattice QCD

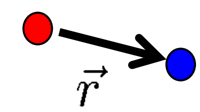






Extension from 2NF ➔ 3NF

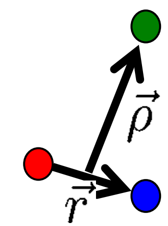
- **In the case of 2N system…**
  - Calc 4pt func ➔NBS amp.



$$\psi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r};t) N(\vec{x};t) | 2N \rangle$$

- **Extension to 3N system**
  - Calc 6pt func ➔NBS amp. of 3N

$$\psi(\vec{r},\vec{\rho}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) N(\vec{x}+\vec{r}/2+\vec{\rho}) | 3N \rangle$$

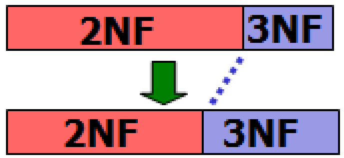


  - Obtain 3NF through

$$(E - H_0^r - H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\sum_{i<j} V_{ij}(\vec{r}_{ij}) + V_{3NF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

by 2N calc

- The combination of (2NF, 3NF) ➔ observables
  - ➔ systematic determination by Lat QCD



The Challenges

- (1) S/N issue

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

  - Use time-dep HAL QCD method

    w/ energy-indep potential

    (➔ no ground state saturation is necessary)

- (2) Computational cost issue

Challenges in 3NF calculation

- Enormous computational cost for correlators
  - # of Wick contraction (permutation)

$$N_{\rm perm} = N_u! \times N_d! \sim [(\tfrac{3}{2}A)!]^2$$ for mass number A

(🡐 **a factor of 2^A speedup** by inner-baryon exchange)

  - # of color / spinor contractions

$$N_{\rm loop} = 6^A \cdot 4^A$$

(color) (spinor)

(🡐 **a factor of 2^A speedup** by "half-spin" method)

$$N = \epsilon_{abc}(q^T C\gamma_5 q)q$$

  - Total cost: $N_{\rm perm} \times N_{\rm loop}$
    - ^{2}H : 9 x 144 = 1 x 10^3
    - ^{3}H : 360 x 1728 = 6 x 10^5
    - ^{4}He : 32400 x 20736 = 7 x 10^8

c.f. T.Yamazaki et al., PRD81(2010)111504
$N_{\rm perm} = 1107$ for ^{4}He in the isospin limit

See also TD (HAL QCD Coll.) PoS LAT2010, 136

Solution: Unified contraction algorithm

TD, M.Endres, arXiv:1205.0585, CPC184(2013)117

- Traditional algorithm

color/spinor contractions (ξ')

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1, \cdots, \xi'_6)$$

Permutations

color ϵ_{abc}, spinor $(C\gamma_5)$, etc.

- New algorithm

[impose the same spacial label at source]

  – Permutation applies to color/spinor indices at “Coeff”

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1, \cdots, \xi'_6)$$

Sum over color/spinor unified list

  – **Permutation DONE beforehand**
    - (Wick contraction and color/spinor contractions are unified)
  – Significant improvement

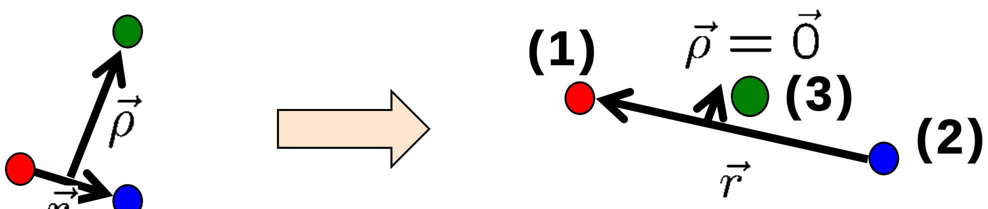
$\times 192$ for ^{3}H/^{3}He, $\times 20736$ for ^{4}He, $\times 10^{11}$ for ^{8}Be

(x add'l. speedup)

3NF calculation in Lat QCD

- We fix the geometry of 3N (← this is not an approximation)
- We study **linear setup**

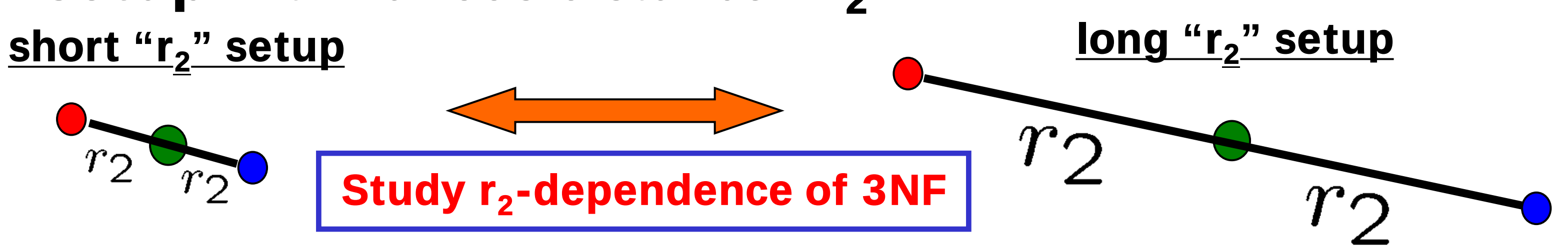
We consider Triton channel



$(\vec{r}_2 \equiv \vec{r}/2)$

  - → $L^{(1,2)\text{-pair}} = L^{\text{total}} = 0$ or 2 only
  - → **Bases are only three**, labeled by **$^1S_0$, $^3S_1$, $^3D_1$ for (1,2)-pair**

- **Linear setup** with various distance **"r_2"**



Extraction of Genuine 3NF

- **Genuine 3NF** can be extracted from **3x3 coupled channel**
  - Both of parity-even 2NF and parity-odd potential required

$$\hat{H}_0 \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} + \left(\underset{(V_{2N}+V_{3NF})}{V} \right) \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} = E \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix}$$

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

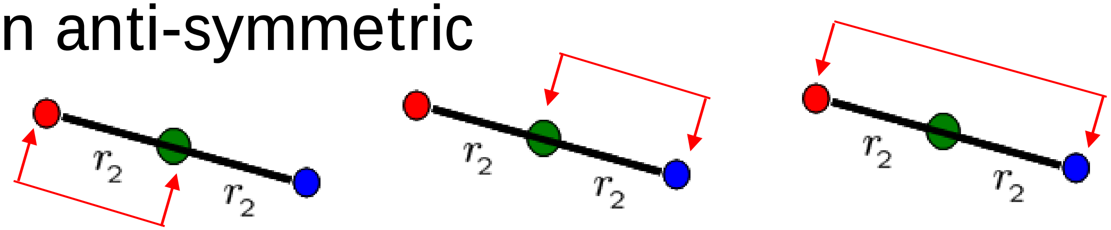
$V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}$: (P = odd)

Target to be determined

- S/N : parity-even 2NF > parity-odd 2NF in Lat QCD
  - ➔ **Desirable to extract 3NF w/ parity-even 2NF only**

Solution using "symmetric" wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric

$\psi_S =$ 

  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
  - Bases are rotated as $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right) \qquad |\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

All pair P=even

$$\hat{H}_0 \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \hat{V}_{3NF} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} = E \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix}$$

No V(P=odd)

Explicit formula for the potential matrix

- The potential matrix for the 2N part in 3x3 coupled channel in linear setup can be written as:

$$V_{2N} = \begin{pmatrix} \begin{array}{c} +V_C^{10}(r)+V_C^{01}(r) \\ +\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} +\frac{1}{2}V_C^{10}(r)-\frac{1}{2}V_C^{01}(r) \\ -\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} -2V_T^{01}(r) \\ +2V_T^{01}(2r) \end{array} \\ \begin{array}{c} +\frac{1}{2}V_C^{10}(r)-\frac{1}{2}V_C^{01}(r) \\ -\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} +\frac{3}{4}V_C^{00}(r)+\frac{1}{4}V_C^{10}(r)+\frac{1}{4}V_C^{01}(r)+\frac{3}{4}V_C^{11}(r) \\ +\frac{1}{2}V_C^{10}(2r)+\frac{1}{2}V_C^{01}(2r) \end{array} & \begin{array}{c} +V_T^{01}(r)-3V_T^{11}(r) \\ +2V_T^{01}(2r) \end{array} \\ \begin{array}{c} -2V_T^{01}(r) \\ +2V_T^{01}(2r) \end{array} & \begin{array}{c} +V_T^{01}(r)-3V_T^{11}(r) \\ +2V_T^{01}(2r) \end{array} & \begin{array}{c} +\frac{1}{2}V_C^{01}(r)+\frac{3}{4}V_C^{11}(r)-V_T^{01}(r)-3V_T^{11}(r) \\ +V_C^{01}(2r)-2V_T^{01}(2r) \end{array} \end{pmatrix} \quad (4.2)$$

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

$V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1}$: (P = odd)

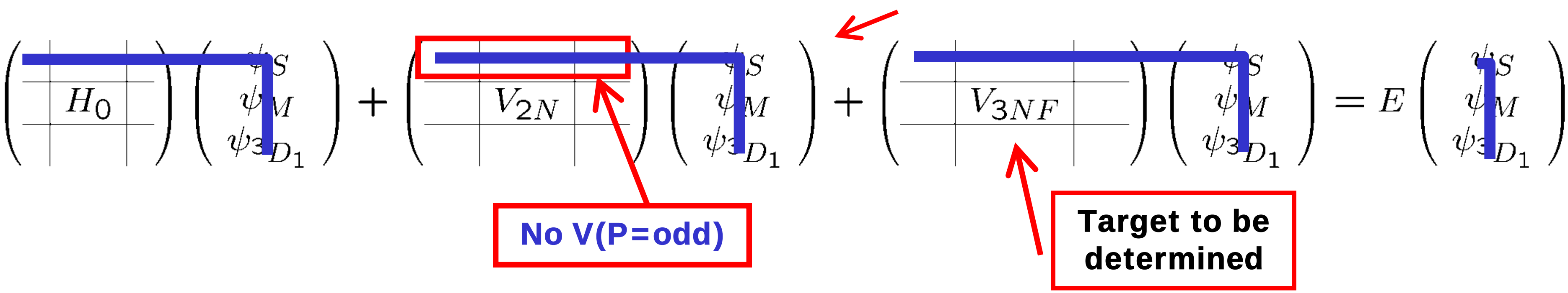
No V(P=odd)

(LO-terms)

$(r_2 \to r$ convention$)$

Solution using "symmetric" wave function

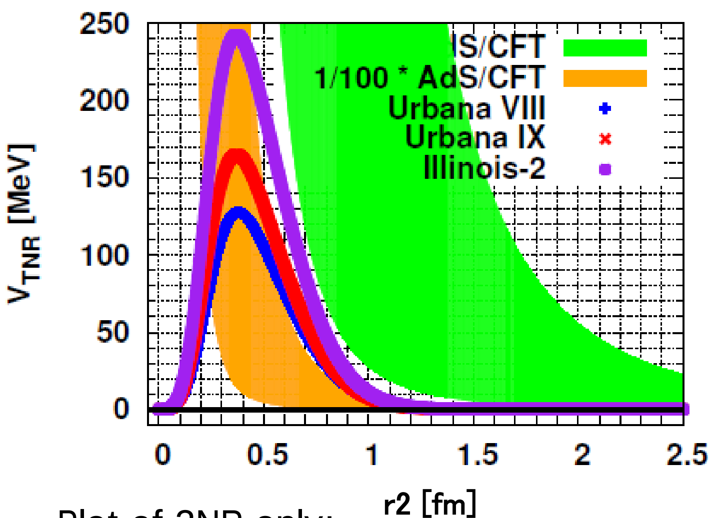
- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric
  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- 3x3 coupled channel is reduced to
  - **<u>one channel with only 3NF unknown</u>**
  - two channels with $V_C^{I,S=0,0}$, $V_C^{I,S=1,1}$, $V_T^{I,S=1,1}$,(3NF) unknown



- ➔ Even without parity-odd V, we can determine one 3NF
  - This method works for any fixed 3D-geometry other than linear

Short-Range 3NF

- We determine 3NF effectively represented by a scalar/isoscalar functional form
  - c.f. phenomenological 3NF to reproduce saturation point of nuclear matter, etc.



Plot of 3NR only:
there is cancellation from 3NA

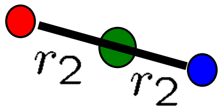
$$V_{3NF} = V_{2\pi E} + (V_{3\pi R}) + V_{3NR}$$

Urbana/Illinois

$$V_{3NR} = U_0 \sum_{cyc} T^2(r_{12}) T^2(r_{13})$$

$$T(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}\right) \frac{e^{-\mu r}}{\mu r} T_{cut}(r)$$

AdS/CFT: $V_{3NF} = +\text{const.} \cdot \frac{1}{r^4}$



K.Hashimoto, N.Iizuka
JHEP 1011 (2010) 058

Lattice QCD Calculations

Numerical Setup & Results

BG/L, BG/Q @KEK

T2K@Tsukuba

SR16000 @YITP, KEK

「SR16000 モデル XM1」

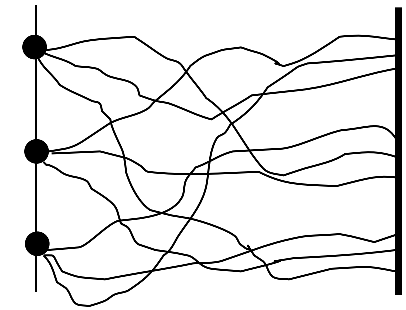
Lattice calculation setup

- Nf=2 clover fermion + RG improved gauge action (CP-PACS)
  - 598 configs x 32 measurements
  - beta=1.95, (a^{-1}=1.27GeV, a=0.156fm)
  - 16^3 x 32 lattice, L=2.5fm
    - M(π) = 1.13GeV
    - M(N) = 2.15GeV
    - M(Δ) = 2.31GeV

(κ(ud) =0.13750)

(Mπ L=14)

CP-PACS Coll. S. Aoki et al., Phys. Rev. D65 (2002) 054505



  - Correlators
    - **Standard nucleon op** to define the wave function / potential **at sink**

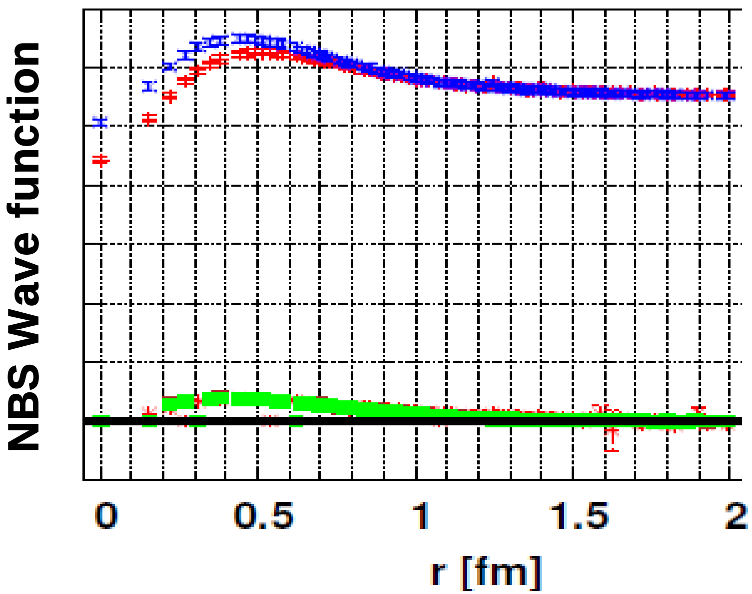
$$N = \epsilon_{abc}(q_a^T C\gamma_5 q_b)q_c$$

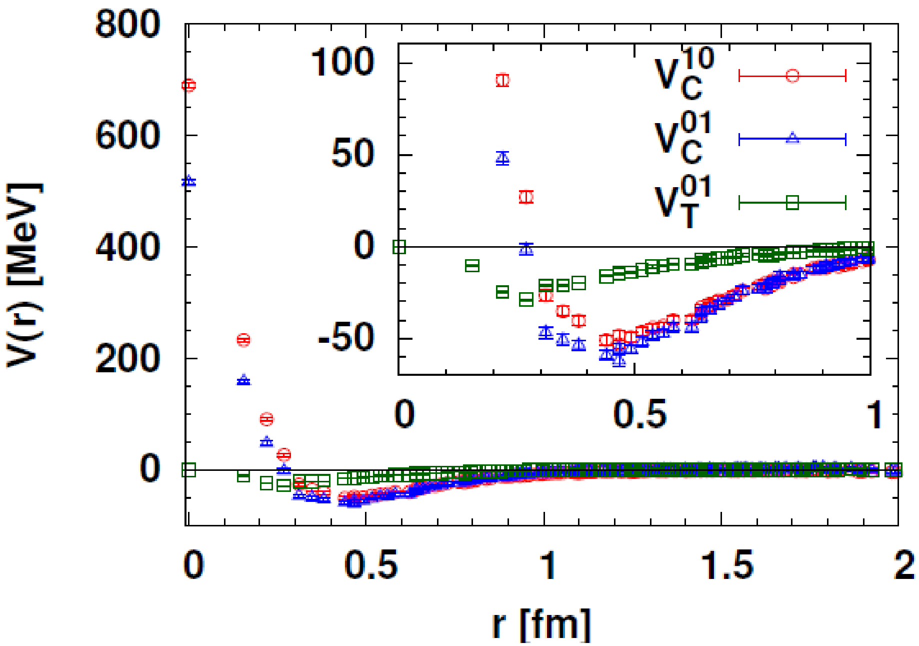
    - **Non-rela limit op** is used to create 3N state **at source**

$$G(\vec{r}_2, t-t_0) = \sum_{\vec{x}} \langle 0 | \underbrace{N(\vec{x}+\vec{r}_2,t)N(\vec{x}-\vec{r}_2,t)N(\vec{x},t)}_{\text{sink}} \underbrace{\overline{NNN}(t_0)}_{\text{source}} | 0 \rangle$$

See also T.Yamazaki et al., PRD81(2010)111504

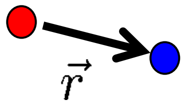
2NF (parity-even) from Lat QCD



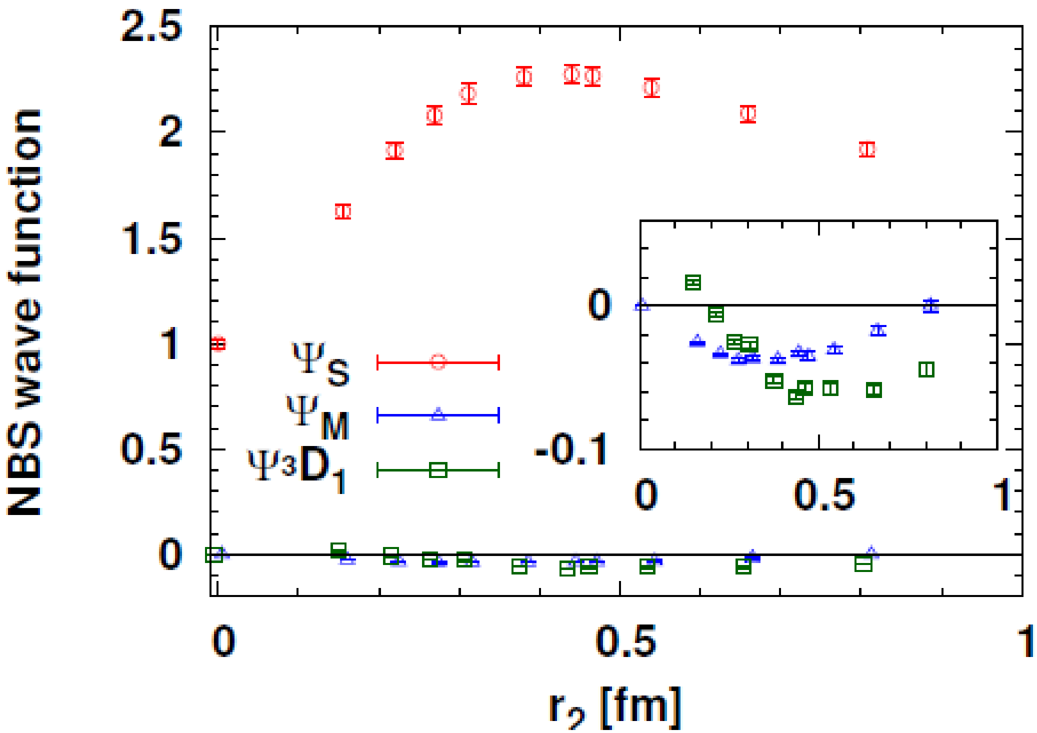


Red: $^1S_0$
Blue: $^3S_1$
Green: $^3D_1$

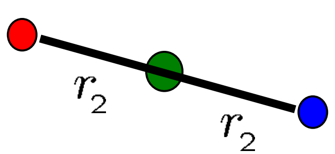
Nf=2, CP-PACS confs, M(π)=1.13GeV



Results for wave functions



T.D. et al. (HAL QCD)
PTP127(2012)723



Red: Ψ_S
Blue: Ψ_M
Green: Ψ_{3D1}

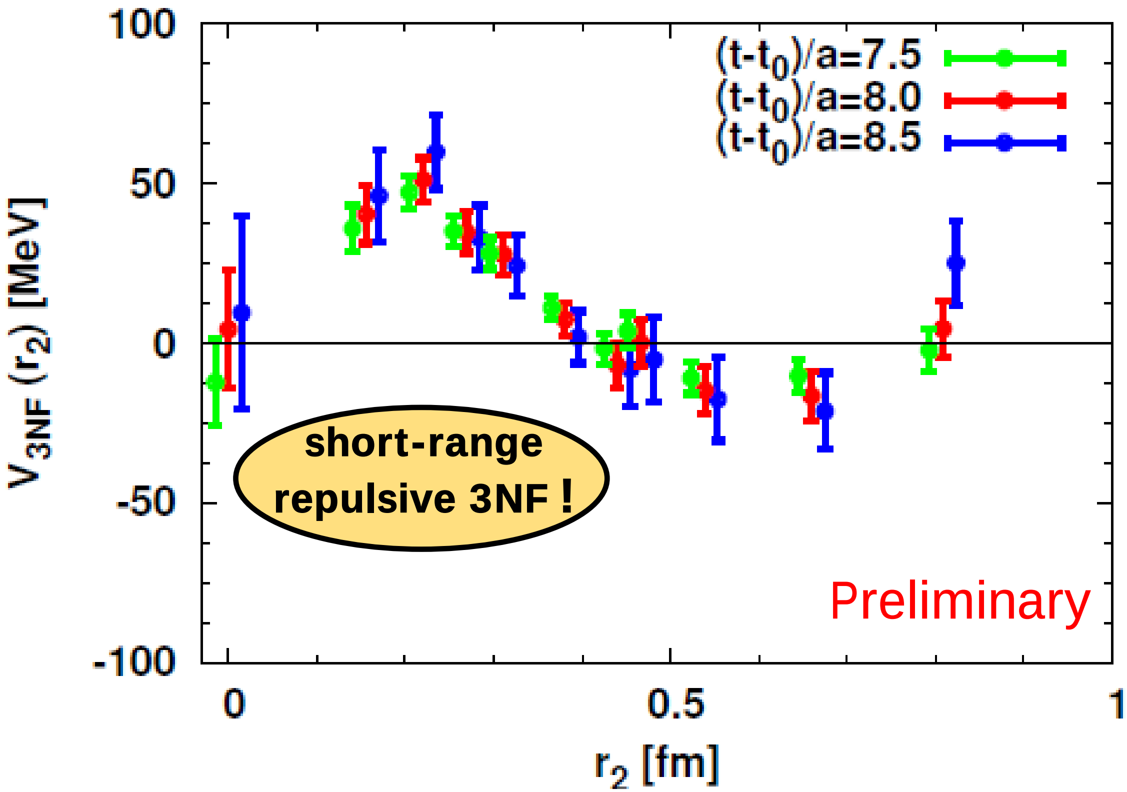
Ψ_S overwhelms the wave function:

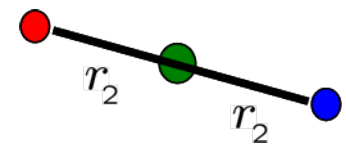
➔ Indication of the dominance of all S-wave component, higher waves suppressed

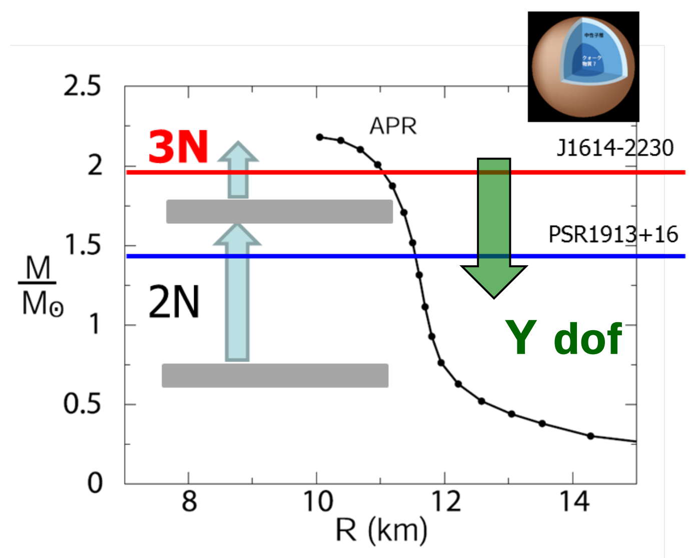
3N-forces (3NF) on the lattice

T.D. et al. (HAL QCD Coll.) PTP127(2012)723

+ t-dep method updates







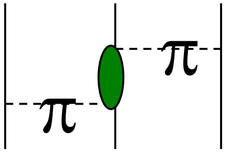
Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=1.1GeV, m_N=2.1GeV

How about other geometries ?

How about YNN, YYN, YYY ?

What is the origin of Lat 3NF ?

- 2πE-type 3NF (Fujita-Miyazawa) is unlikely
  - Strongly suppressed by m_π = 1.13GeV



- It may be attributed to quark/gluon dynamics directly
  - Recall generalized 2BF in SU(3)f …

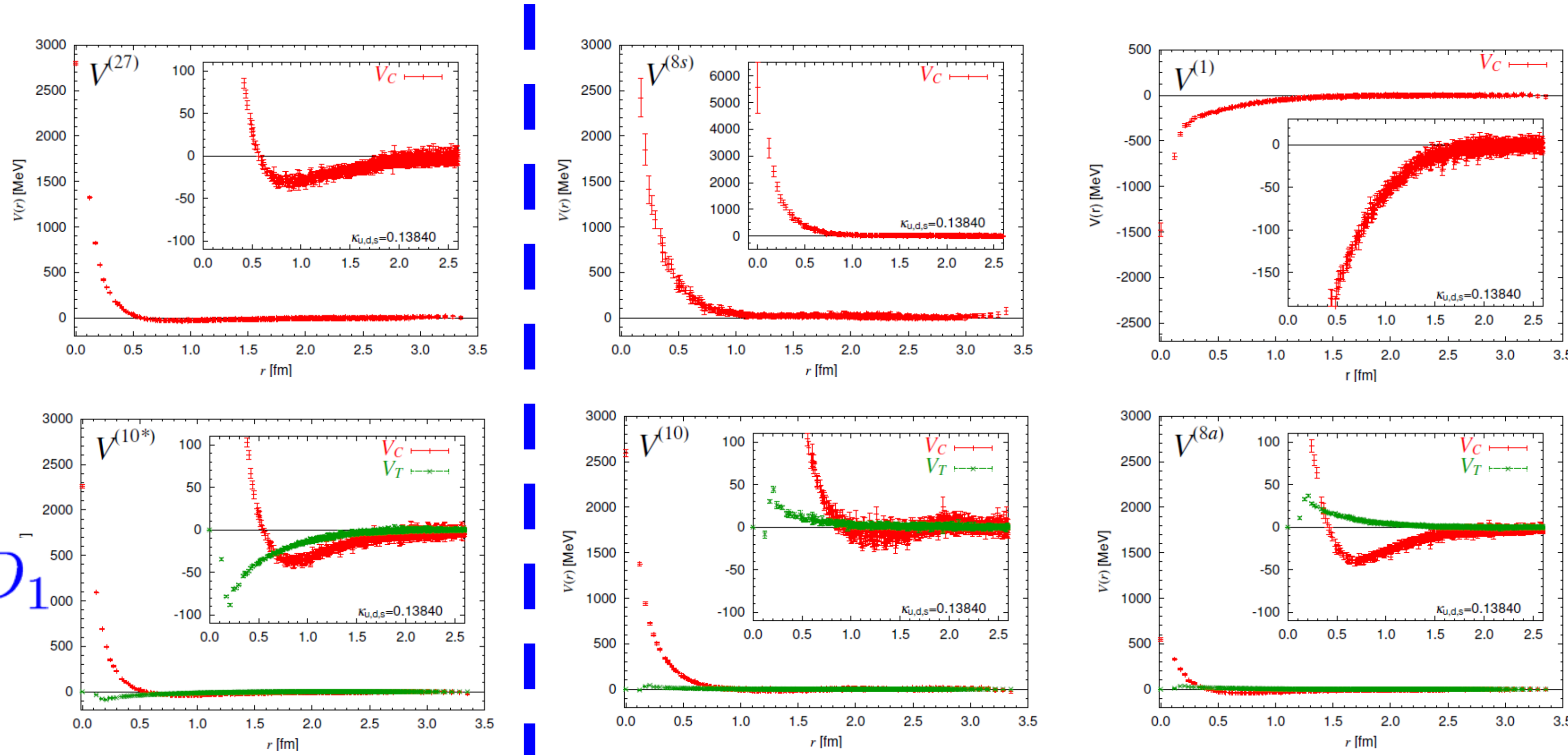
SU(3) study

BB potentials

a=0.12fm, L=3.9fm, m(PS)= 0.47-1.2GeV

$^1S_0$

$^3S_1-^3D_1$



27,10*:
Same as NN

8s,10:
strong repulsive core

1s: deep attractive pocket
8a: weak repulsive core

Repulsive core
← Pauli principle !

M.Oka et al., NPA464(1987)700

T.Inoue et al. (HAL QCD Coll.), NPA881(2012)28

Also seen in SU(2)c , Takahashi et al.,, PRD82(2010)094506
Charmonium-N, Kawanai-Sasaki, PRD82(2010)091501
Meson-baryon, Y.Ikeda et al., arXiv:1111.2663

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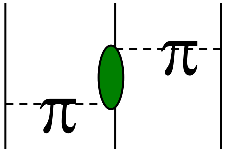


- It may be attributed to quark/gluon dynamics directly
  - Recall generalized 2BF in SU(3)f …
    - ➔ Pauli principle works well
  - What will be the Pauli-principle effect in 3NF from a viewpoint of the Quark Model ?
  - c.f. OPE (pert. QCD) predicts repulsive 3NF at short distance

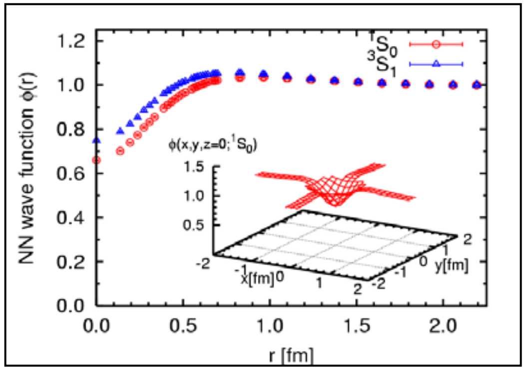
S.Aoki et al., arXiv:1112.2053

Summary

Our Approach [HAL QCD method]

Lattice QCD

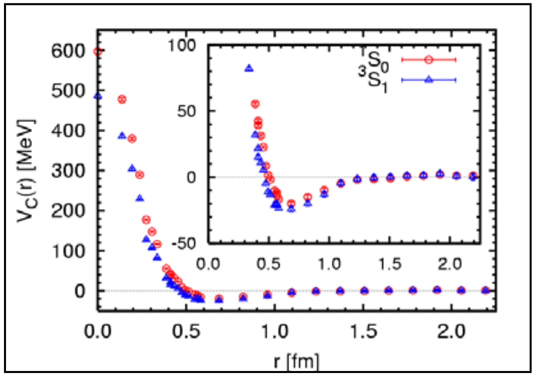
NBS wave func.



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle$$
$$\simeq e^{i\delta(k)} \sin(kr + \delta(k))/(kr)$$

(at asymptotic region)

Lat Nuclear Force



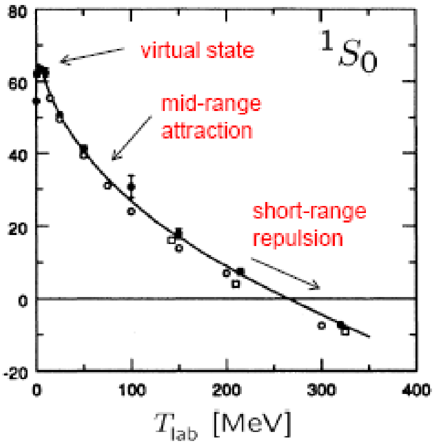
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,')$$

Lat potential is faithful to phase shift by construction

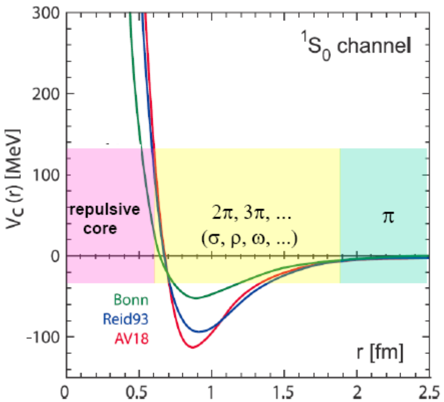
Analog to …

Scattering Exp.

Phase shifts

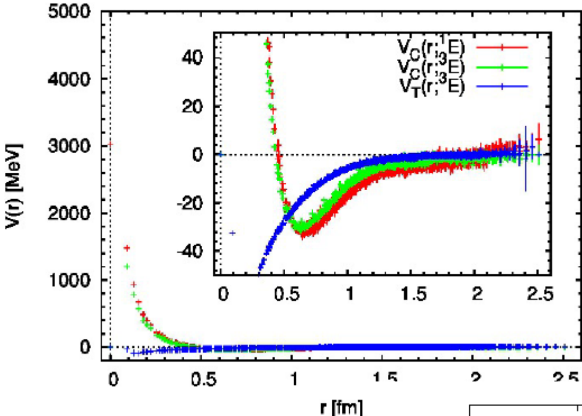


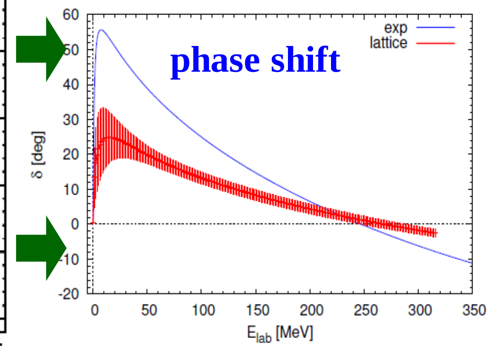
Phen. Potential



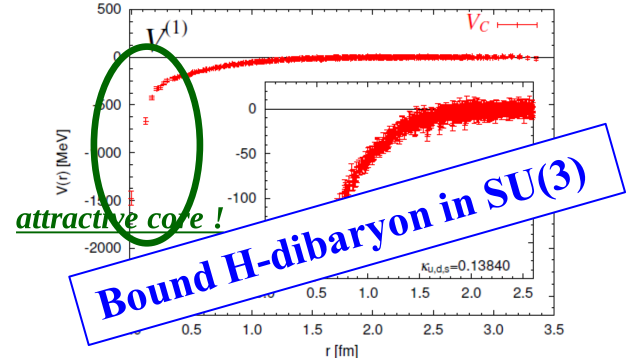
Research Highlight

NN forces (central & tensor)

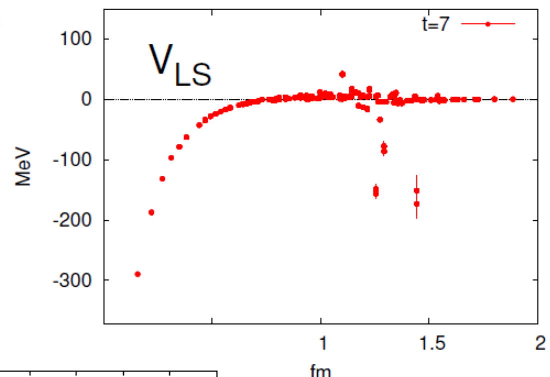




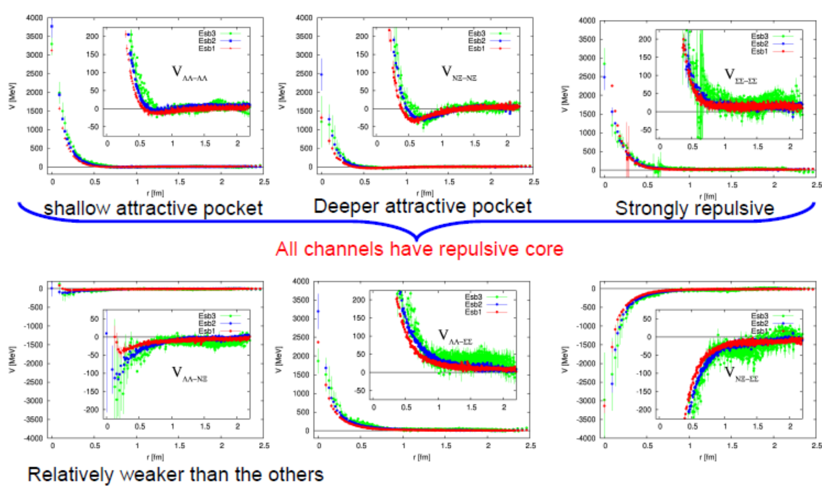
Hyperon Forces

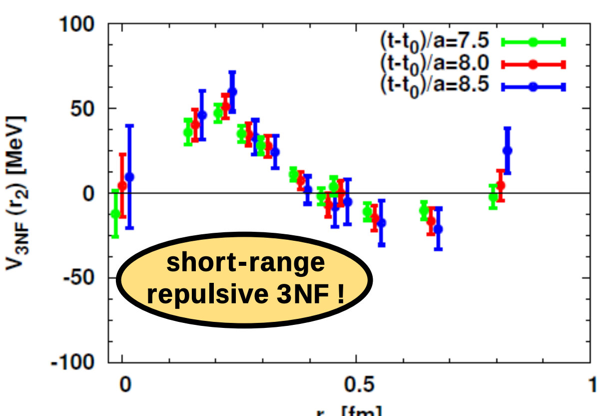


P-odd forces, LS force

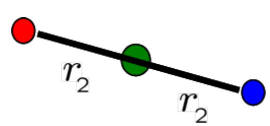


Coupled channel beyond SU(3)





Three-Nucleon Forces

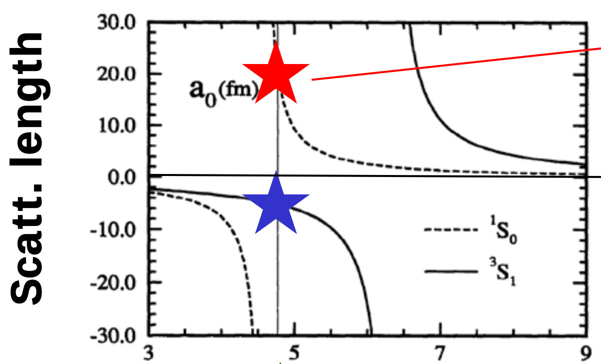


Future Prospects
in Lattice Nuclear Forces

- *What is most important next ?*

Towards realistic potential on the Lattice

- **Physical mass point**, **Infinite V limit**, **continuum limit**
  - Physical $m\pi$ crucial for OPEP, chiral extrapolation won't work



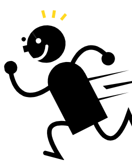
m_q

Y.Kuramashi, PTPS122(1996)153

Phys. point

$a_0(^1S_0) \sim 20\text{fm}$

"Unitary Region"



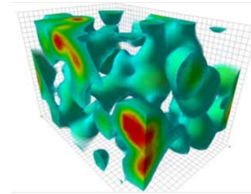
We are here

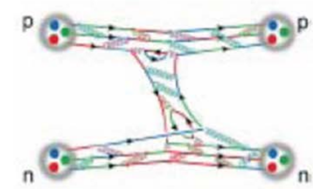
$\rightarrow m_q$

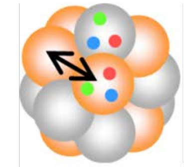
  - Gauge confs generation at $m\pi$ = 140MeV, L=~10fm on the K computer

K computer

Summary and Prospects







- Hadron Interactions by 1st principle Lat calc
  - Bridging different worlds:
    Particle Physics / Nuclear Physics / Astrophysics

- Lattice QCD results for NN, **YN/YY**, NNN
  - Intriguing physics even at heavy quark masses

- **Next major step:** physical quark mass point !
  - Breakthroughs in S/N issue & Comput. cost issue

Thermodynamic limit & continuum limit

YOUR new idea
Welcome !

➔ Realistic hadron interactions
➔ Nuclear Physics on the Lattice !