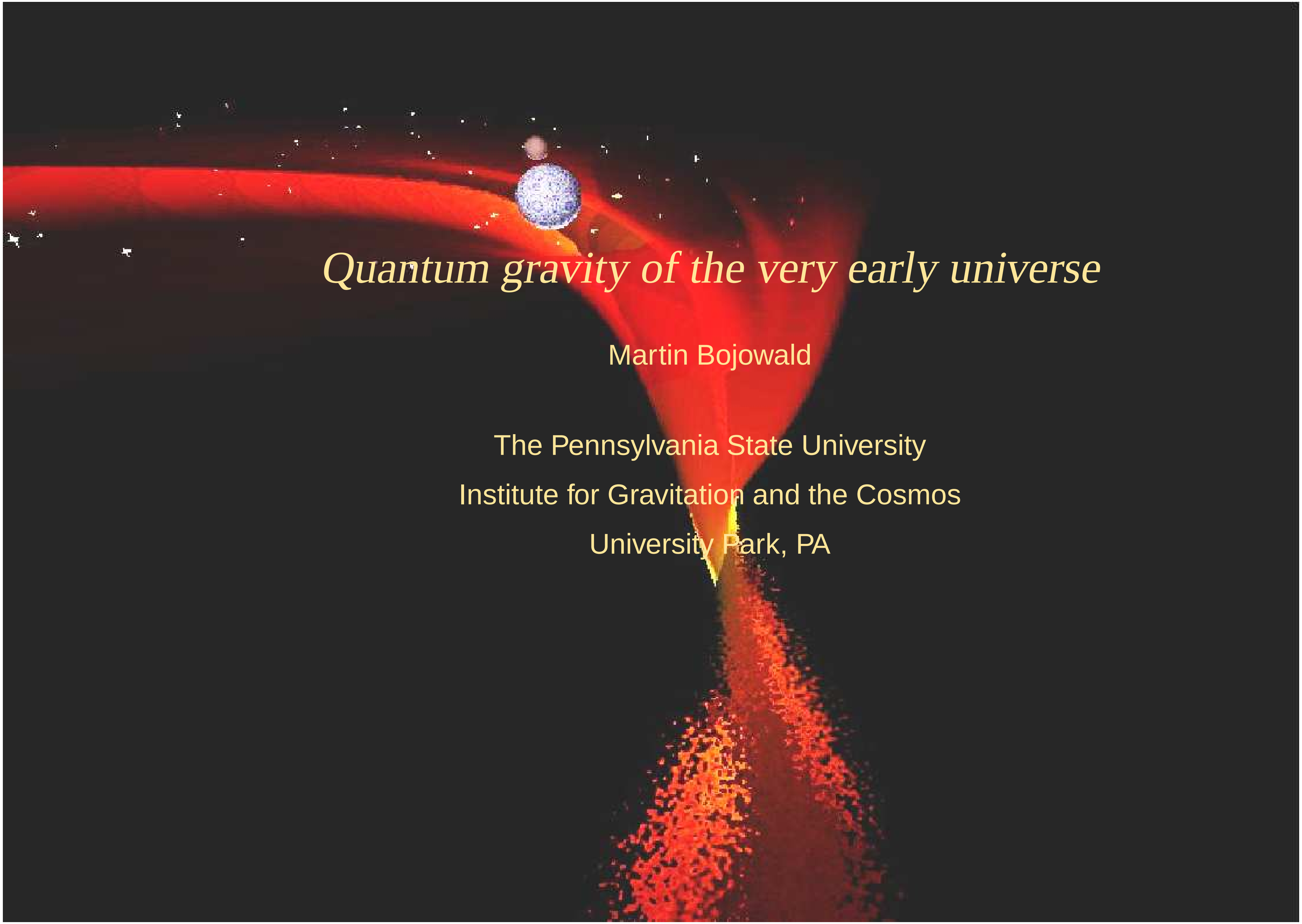

Quantum gravity of the very early universe

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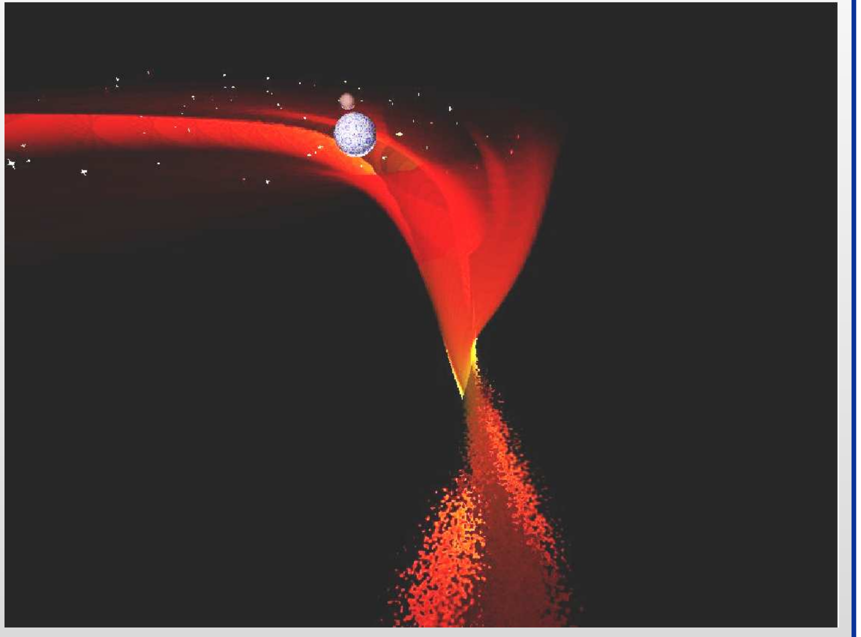
Quantum gravity

Processes in the very early universe require

- → general relativity (expansion of space),
- → quantum physics (hot, dense).



Sometimes, this even involves quantum physics of gravity.

Described by the geometry of space-time; quantize space-time.

One possible consequence:
elementary constituents, “atoms of space.”

Scales

Dimensional arguments to estimate direct effects:

Unique length parameter, the Planck length
$\ell_{\rm Pl} = \sqrt{G\hbar/c^3} \approx 10^{-35}{\rm m}$

and mass parameter, the Planck mass
$M_{\rm Pl} = \sqrt{\hbar c/G} \approx 10^{18}{\rm GeV} \approx 10^{-6}{\rm g}$.

Quantum gravity inevitable at Planck density $\rho_{\rm Pl} = M_{\rm Pl}/\ell_{\rm Pl}^3$.

About a trillion solar masses in the region of the size of a single proton.

(Current density of the universe: about an atom per cubic meter.)

PENNSTATE

Indirect evidence

1905, Albert Einstein: Analysis of Brownian motion as convincing evidence for atoms.

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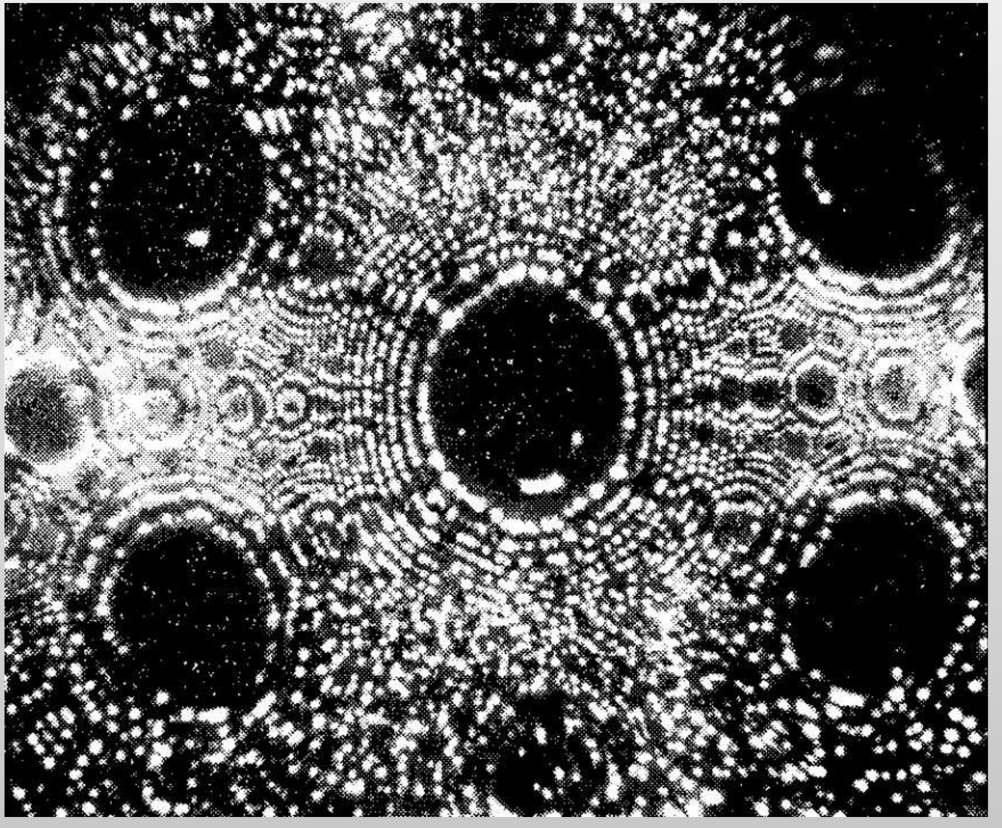
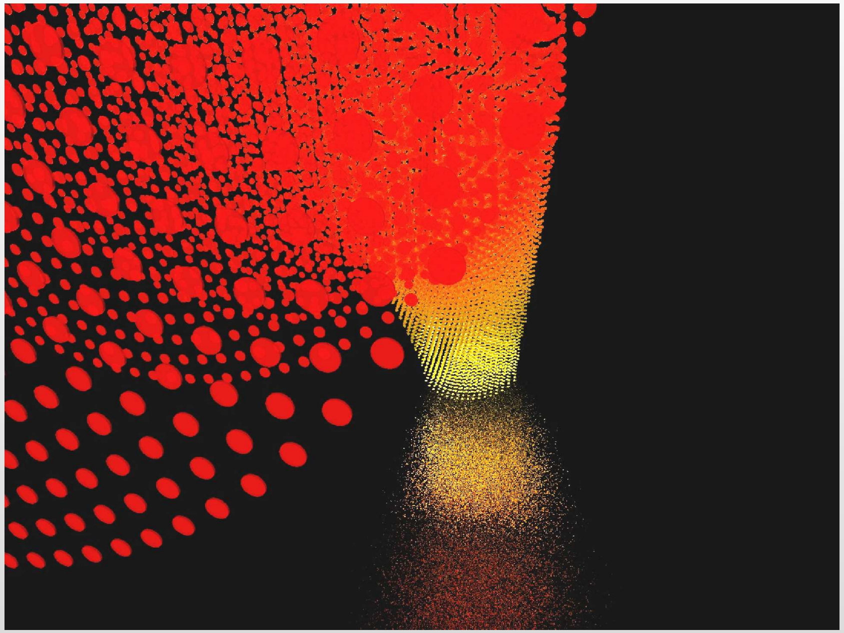


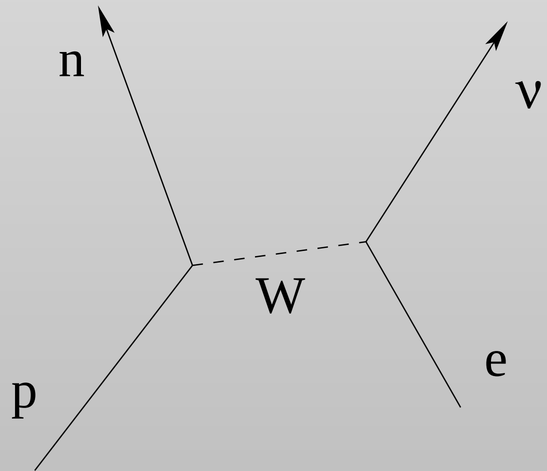
1955, Erwin Müller: First direct image of atoms using field ion microscopy.

Expansion

An expanding discrete space grows not continuously but atom by atom.

Implications weak for a large universe, but may be noticeable by sensitive measurements.





Example: Abundances of light elements depend on baryon-photon ratio during big-bang nucleosynthesis (proton-neutron interconversion by weak interaction).

Baryon-photon ratio depends on dilution behavior of radiation and (relativistic) fermions.

Standard model of cosmology

Big bang: Planckian density, classical singularity (13.8 billion years ago).

Inflation: Accelerated expansion at energy scale $\sim 10^{-10}\rho_{\rm Pl}$.

Particle production (cosmological Schwinger effect).

Seeds for matter distribution as seen in cosmic microwave background (CMB) and galaxies.

Baryogenesis: Baryons form, matter/antimatter asymmetry.

Nucleosynthesis: Nuclei form (about 75% hydrogen and deuterium, 25% helium, trace amounts of other light elements).

CMB release: Atoms neutralize, universe becomes translucent (after 380,000 years).

Big-bang singularity

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \quad , \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

(scale factor a, energy density ρ, pressure P) implies

$$\dot{\mathcal{H}} = -\frac{4\pi G}{3}(\rho + 3P) - \mathcal{H}^2$$

for Hubble parameter $\mathcal{H} = \dot{a}/a$.

With strong energy condition $\rho + 3P \geq 0$:

$\dot{\mathcal{H}} \leq -\mathcal{H}^2$ or $\mathrm{d}\mathcal{H}^{-1}/\mathrm{d}t \geq 1$ and $\mathcal{H}^{-1} \geq \mathcal{H}_0^{-1} + t - t_0$.

If $\mathcal{H}_0^{-1}$ negative, $\mathcal{H}^{-1}$ positive at $t_1 = t_0 - \mathcal{H}_0^{-1}$; $\mathcal{H}^{-1} = 0$ at some time, when $\mathcal{H} \to \infty$, $\rho \to \infty$. Past singularity if expanding.

Singularity theorems:
singularities generic in space-time dynamics.

Shortcomings of the standard model

→ Singularity unphysical.

→ Initial vacuum state appropriate?
Matter/antimatter asymmetry difficult to explain.

→ If there was a prehistory of the quantum universe before the big bang, more time existed for asymmetry to build up.

→ Matter equation of state important for some aspects of transition.

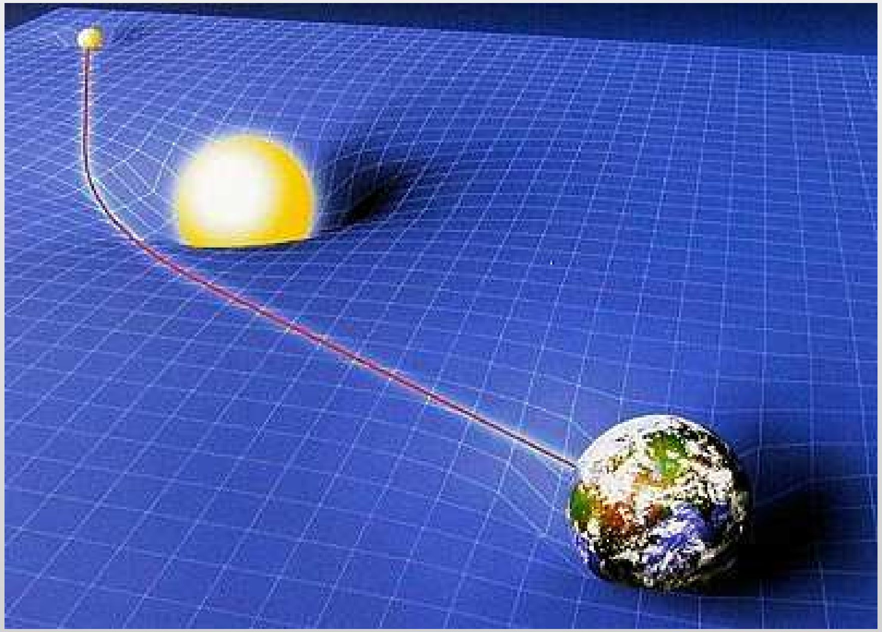
Need more information about quantum gravity, space-time structure.

Constructing quantum gravity

Gravity is “strongly interacting” at a fundamental, non-perturbative level.

Non-renormalizability: cannot be quantized as weakly-interacting theory of gravitons.

Well-known weak form of gravity as long-range remnant of more elementary theory.



Different approaches, no fully consistent version yet.

Quantization directly addressing structure of space and time: loop quantum gravity. (Background independence.)

Floating lattices

Theory can be constructed by means analogous to lattice QCD, but with one crucial difference:

General covariance implies that all states must be invariant under deformations of space (coordinate changes).

- $\rightarrow$ Regular lattices too restrictive (instead “floating”).
- $\rightarrow$ No well-motivated restriction on valence of lattice vertices (except simplicity).
- $\rightarrow$ Lattice edges may be knotted and interlinked.
- $\rightarrow$ Superpositions of different lattice states.
- $\rightarrow$ States of continuum theory described by lattices; no approximation, no continuum limit.

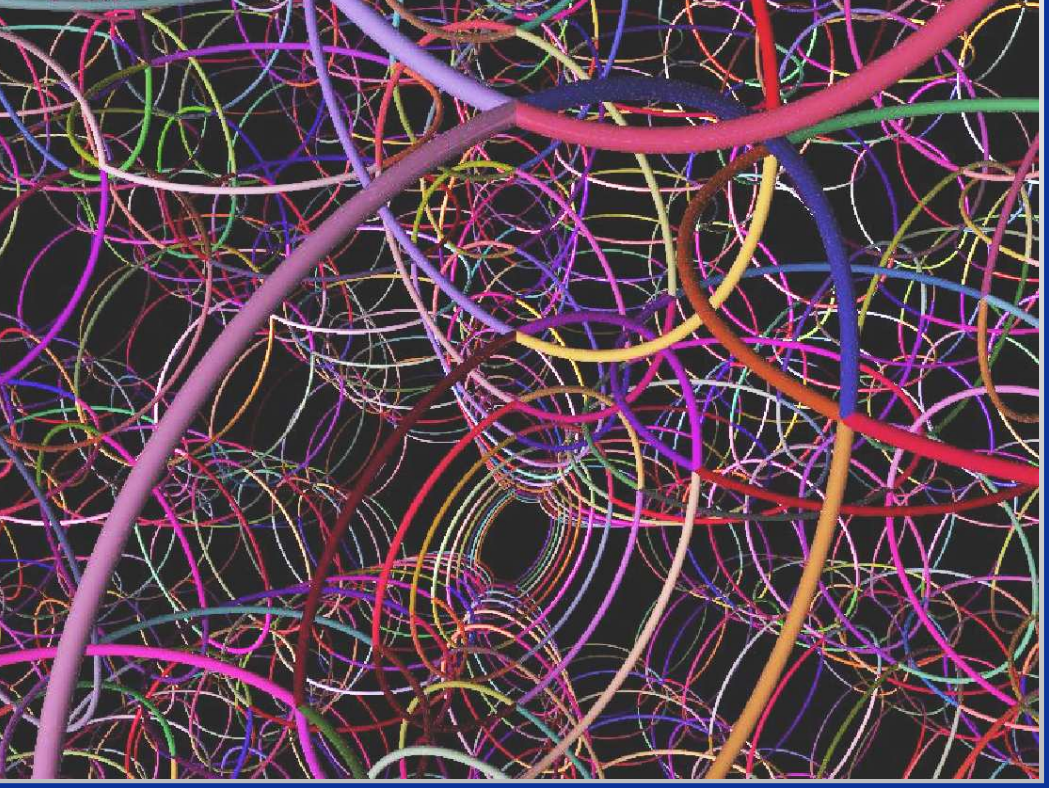
  (Alternative viewpoint: Causal Dynamical Triangulations.)

Interactions

Fundamental lattice theory for quantum geometry. Geometrical excitations generated by creation operators for lattice links.

Near continuum: Highly excited many-particle states, “interacting”.

Resulting physics mainly analyzed in model systems.



Loop quantum gravity

Describe space-time geometry by su(2)-valued "electric field" $\vec{E}_i$ and "vector potential" $\underset{\rightarrow}{A}_i$ (Ashtekar–Barbero variables).

Electric field: triad, determines spatial distances/angles by three orthonormal vectors $\vec{E}_i$, $i = 1, 2, 3$, at each point in space.

Vector potential: $\underset{\rightarrow}{A}_i = \underset{\rightarrow}{\Gamma}_i + \gamma \underset{\rightarrow}{K}_i$ with $\underset{\rightarrow}{\Gamma}_i$ related to intrinsic curvature of space, $\underset{\rightarrow}{K}_i$ to extrinsic curvature in space-time (γ: real parameter).

$\vec{E}_i$ as momentum of $\underset{\rightarrow}{A}_i$: $\{\underset{\rightarrow}{A}_i(x), \vec{E}_j(y)\} = 8\pi\gamma G \delta_{ij} \underset{\rightarrow}{\vec{\delta}}\, \delta(x, y)$.

Proceed by canonical quantization, observing special properties due to symmetries of the theory: general covariance.

Lattice states

Holonomies $h_e = \mathcal{P}\exp(\int_e \mathrm{d}\lambda \underset{\rightarrow}{A}_i \cdot \vec{t}_e \tau^i)$ for Ashtekar connection $\underset{\rightarrow}{A}^i$ (curvature), spatial curves e; $\tau^j = \frac{1}{2} i \sigma^j$ with Pauli matrices.

Define basic state ψ_0 by $\psi_0(\underset{\rightarrow}{A}_i) = 1$: independent of connection.

Excited states, simplified U(1)-example where $h_e = \exp(i \int_e \mathrm{d}\lambda \underset{\rightarrow}{A} \cdot \vec{t}_e)$:

$$\psi_{e_1,k_1;\ldots;e_i,k_i} = \hat{h}_{e_1}^{k_1} \cdots \hat{h}_{e_i}^{k_i} \psi_0$$

General state labeled by graph g and integers k_e as quantum numbers on edges

$$\psi_{g,k}(\underset{\rightarrow}{A}) = \prod_{e \in g} h_e(\underset{\rightarrow}{A})^{k_e} = \prod_{e \in g} \exp(i k_e \int_e \mathrm{d}\lambda \underset{\rightarrow}{A} \cdot \vec{t}_e)$$

Discrete Geometry

Ashtekar connection has momentum $\vec{E}_i$ such that $\sum_i \vec{E}_i \otimes \vec{E}_i = (\det q) \cdot \vec{\vec{q}}$ gives the spatial metric $\vec{\vec{q}}$.

Flux $\int_S \mathrm{d}^2 y \underset{\rightarrow}{n} \cdot \vec{E}_i$ ($\underset{\rightarrow}{n}$ co-normal to surface S) quantized as derivative operator, measures excitation level:

$$\int_S \mathrm{d}^2 y \underset{\rightarrow}{n} \cdot \hat{\vec{\vec{E}}} \psi_{g,k} = \frac{\gamma G \hbar}{i} \int_S \mathrm{d}^2 y \underset{\rightarrow}{n} \cdot \frac{\delta \psi_{g,k}}{\delta \underset{\rightarrow}{A}(y)} = \gamma \ell_{\mathrm{P}}^2 \sum_{e \in g} n_e \mathrm{Int}(S,e) \psi_{g,k}$$

with intersection number $\mathrm{Int}(S,e)$.

Discrete geometry: for gravity, flux represents spatial metric (area, volume).

Dynamics

Schematic Hamiltonian

$$\hat{H} = \sum_{v,IJK} \epsilon^{IJK} \mathrm{tr}(h_{v,e_I} h_{v+e_I,e_J} h^{-1}_{v+e_J,e_I} h^{-1}_{v,e_J} h_{v,e_K} [h^{-1}_{v,e_K}, \hat{V}])$$

summing over vertices v of graph and triples (IJK) of edges.

Gauge fields via independent type of holonomies.
Fermions as spinors in vertices. Matter Hamiltonian added to $\hat{H}$.

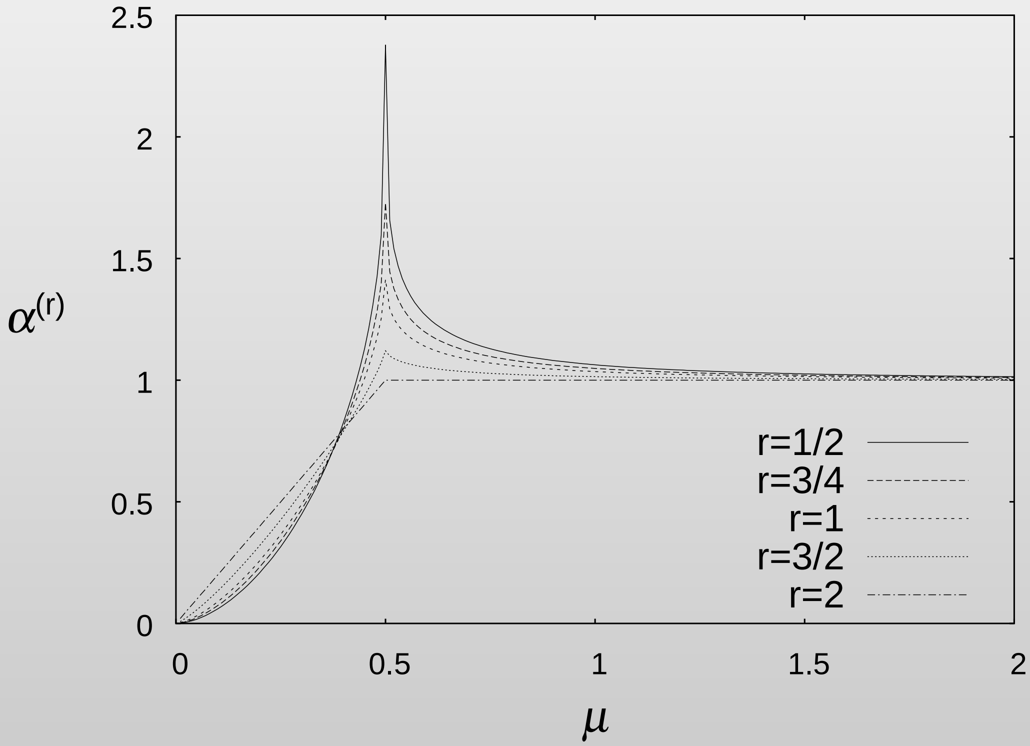
Total Hamiltonian well-defined, no divergences.
But limit of classical space-time poorly understood.

Main challenge for space-time dynamics:
Understand discrete quantum geometry combined with general covariance.

Quantum corrections

⟶ Corrections of inverse metric components in Hamiltonians: flux with discrete spectrum containing zero.

Correction function (with quantization ambiguities, e.g. r).



flux eigenvalues

⟶ Higher-order corrections: holonomies

⟶ Quantum back-reaction (generic)

Harmonic cosmology

Isotropic cosmology: Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

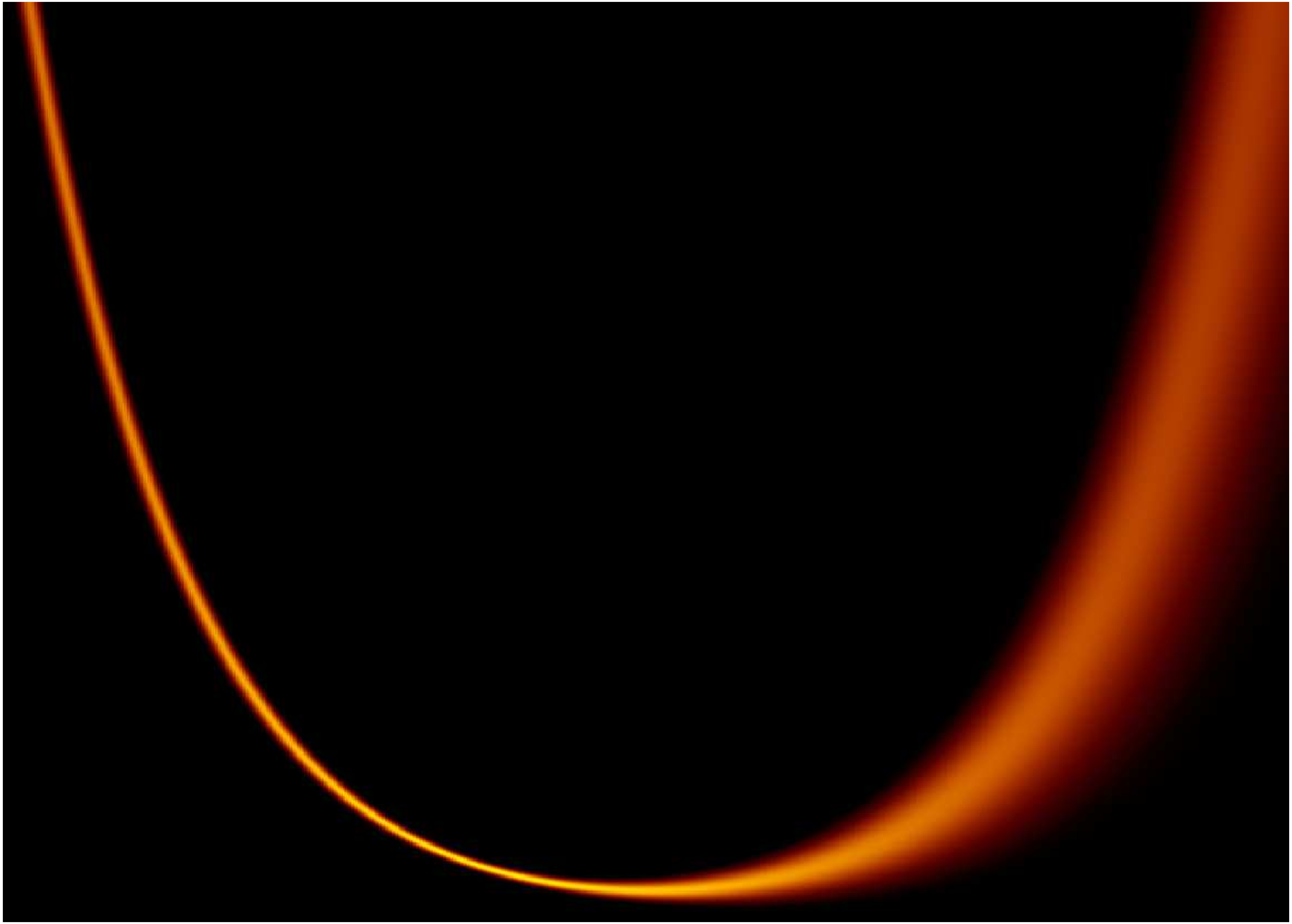
receives corrections by higher powers of $\dot{a}$ ($p_a \to \sin(\delta p_a)/\delta$).

Solvable model for free, massless scalar. Series can be resummed to give

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_0}\right)$$

with ρ_0 of the order of $\rho_{\rm Pl}$.

(Based on $\mathrm{sl}(2,\mathbb{R})$ algebra $[\hat{V},\hat{J}] = i\hbar\hat{H}$, $[\hat{V},\hat{H}] = -i\hbar\hat{J}$, $[\hat{J},\hat{H}] = i\hbar\hat{V}$ with volume $\hat{V}$, $J = V\exp(iV\mathcal{H})$, Hamiltonian $\hat{H}$.)



Implications

Discrete space-time: finite capacity to store energy. Gravity turns repulsive at high densities.

Bounce at about Planck density (probably less) can resolve singularity problem.

Matter properties relevant throughout cosmic evolution.
Bounce cosmology: attempt to provide alternative to inflation to explain nearly scale-free spectrum of anisotropies.

Scale-free for dust matter (vanishing pressure) during collapse. Deviations when quation of state changes.

Exotic matter may help to prevent large anisotropy.

[M. Novello, S. Bergliaffa: Phys. Rep. **463** (2008) 127–213]

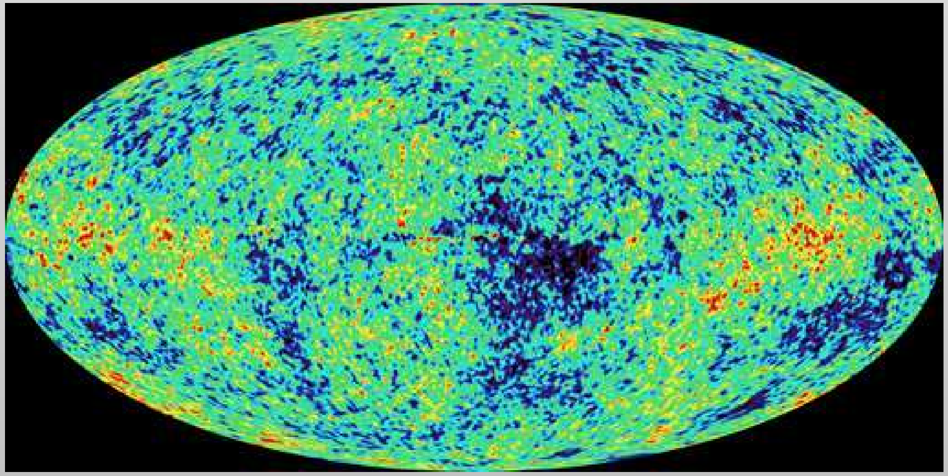
Cosmology

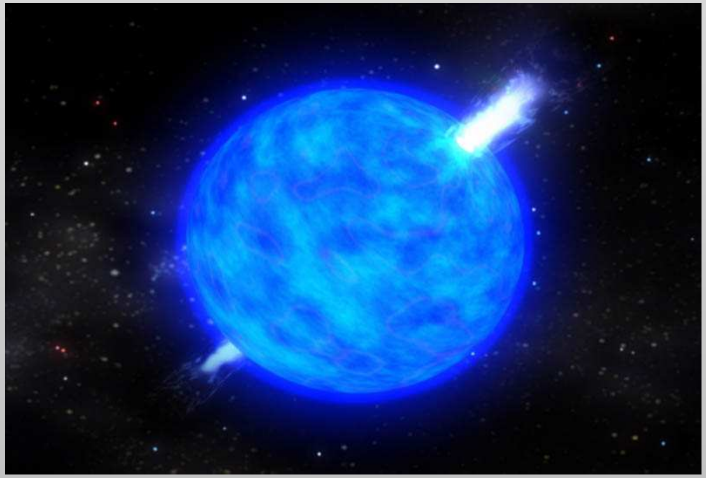
With matter interactions and inhomogeneities: perturbation theory around solvable model.

Indirect effects of atomic space-time: small individual corrections even at high energies, might add up coherently.

$\longrightarrow$ *cosmology,* high energy density, long evolution

$\longrightarrow$ *high energy particles* from distant sources (GRBs).





Big-bang nucleosynthesis

Quantum gravity:

Maxwell and Dirac Hamiltonians subject to different quantum corrections. May change dilution behavior.

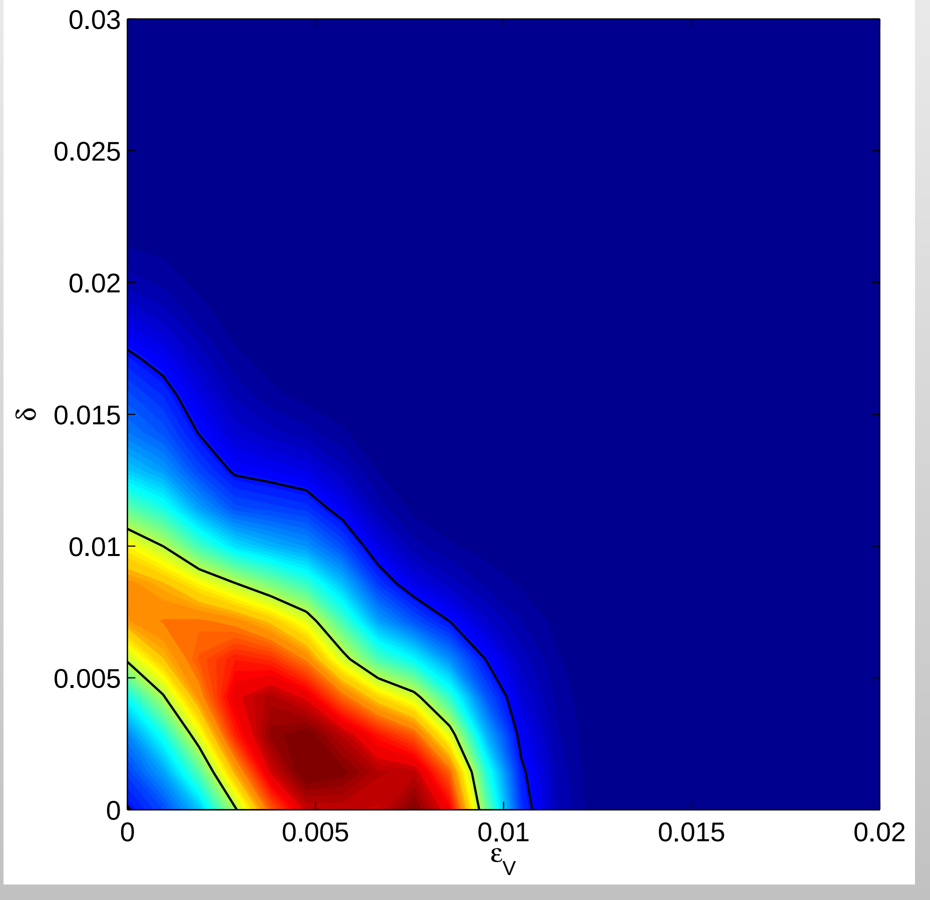
So far: equations of state change in the same way for photons and relativistic fermions. (Related to general covariance.)

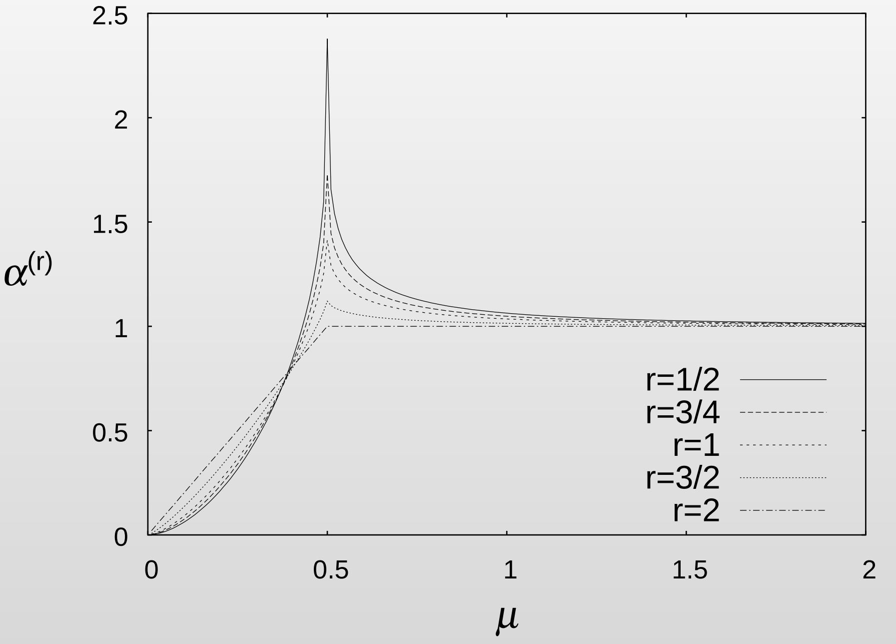
Effects not very strong, but close to being interesting: Upper bound $\rho < 3/\ell_{\mathrm{Pl}}^3$ for density of atoms of space.

However, precision of big-bang nucleosynthesis observations difficult to improve.
More promising: details of cosmic microwave background.

CMB with inverse-volume corrections

Hamiltonians with corrections $\alpha \sim 1 + \delta$ from inverse volume in loop quantum gravity.





δ can be estimated by CMB analysis, so far consistent with zero.

(Combined analysis with slow-roll parameter ϵ_V for behavior of inflation.)

Black holes

General relativity: impossible to stop collapse under very general assumptions on equation of state.

Gravity always attractive, dominant force when matter sufficiently dense.

Quantum gravity: space-time dynamics changes, repulsive gravity at extremely high density.

Non-singular collapse, but still with horizon trapping light (for finite time): black holes.

Horizon Hawking-evaporates, stellar explosion when horizon disappears. Collapse models depend on matter behavior.

Parity

Vector potential defined as

$$\underset{\rightarrow}{A}_i = \underset{\rightarrow}{\Gamma}_i + \gamma \underset{\rightarrow}{K}_i$$

where $\underset{\rightarrow}{\Gamma}_i$ parity-odd, $\underset{\rightarrow}{K}_i$ parity-even.

Unless γ pseudoscalar, non-trivial parity behavior of $\underset{\rightarrow}{A}_i$.

Equations of motion parity invariant classically, but invariance may be broken after replacing $\underset{\rightarrow}{A}_i$ with $h_e(\underset{\rightarrow}{A}_i)$.

May be relevant for baryogenesis.

Also: some bounce models show change of orientation (universe “turns inside out”).

Quantum gravity and the quark-gluon plasma

Still many orders of magnitude from quark-gluon plasma toward the Planck scale, at best indirect consequences.

→ Matter equation of state important for collapse/bounce scenarios:
development of anisotropy and evolution of structure.

→ Cosmological prehistory relevant for baryogenesis: matter/antimatter-symmetric initial state or a more messy one after the collapse of an entire universe?

→ Space-time symmetries fundamental?

Summary

→ Quantum theory of space-time as gauge theory. Crucial new feature: general covariance.
In loop quantum gravity, implies (irregular) lattice structure even for continuum theory.

→ Direct effects important at extremely high density, but indirect effects possible in intermediate regimes.

Then, equation of state of matter required for details.

→ No observation yet, but bounds on theory are becoming interesting.