Models for strong interaction physics

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Outline:

- Discussion of various models used to study strong interaction.
- QCD phase diagram and location of CEP from Polyakov Loop enhanced Nambu-Jona-Lasinio (PNJL) model.

Earliest model

The Quark Model

- Large zoo of *elementary* particles came up by 1960s
- Quantum numbers like mass, charge, baryon number, isospin, strangeness were used to classify them.
- By 1964 realization came (to Gell-Mann and Zweig) that a simple constituent picture of these *elementary* particles is in the offing.
- The fundamental constituents $\rightarrow$ quarks and gluons were finally discovered at Stanford Linear Accelarator Center 1970s.

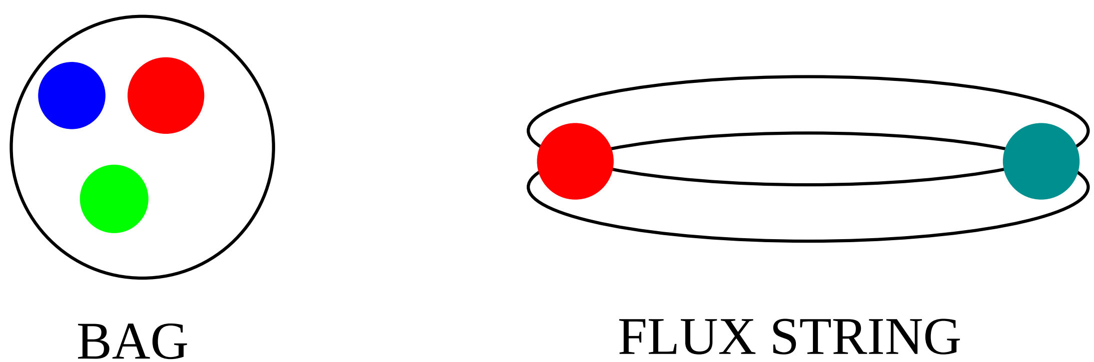
Quark model is by far the most successful model associated with strong interactions.

Confinement model

If quarks are inside the hadrons why don’t they come out?

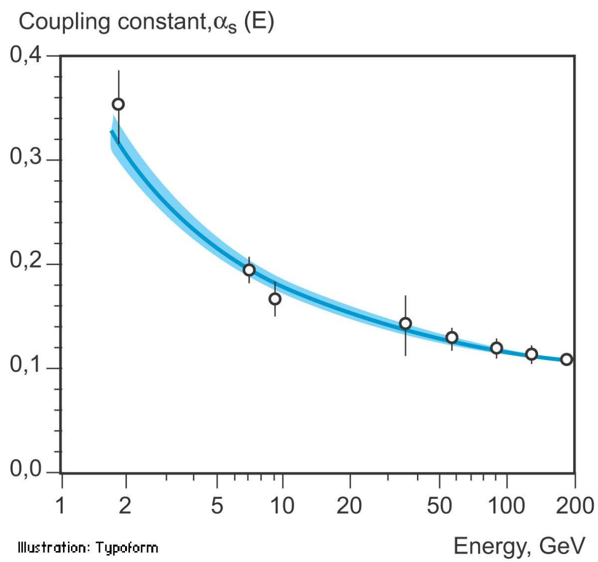
- Bag Model: Quarks are inside a deep potential hole (1960s)
- String Model: Quarks are held by a strong color electric flux tube (1960s)

Both were quite successful in describing the hadron phenomenology.



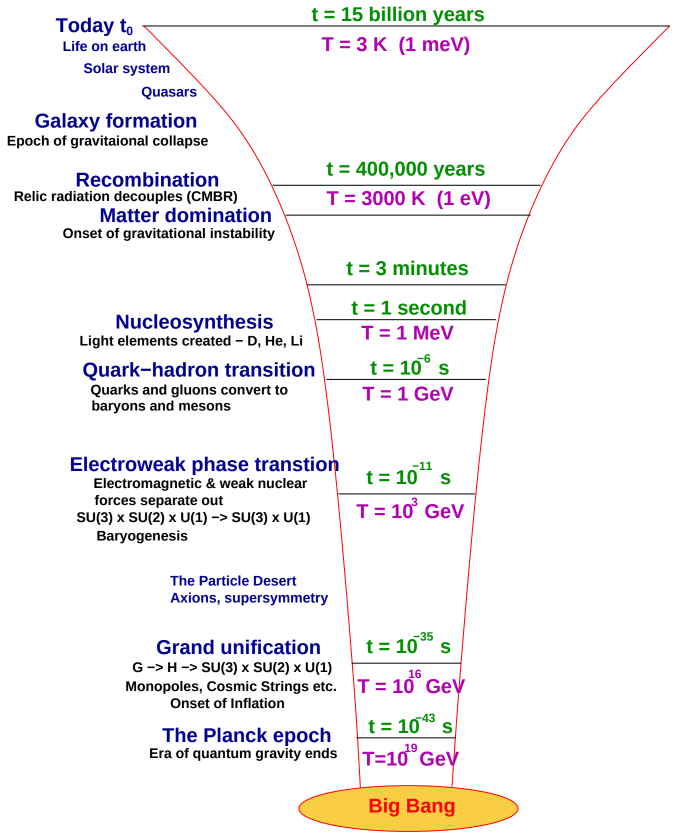
Theory

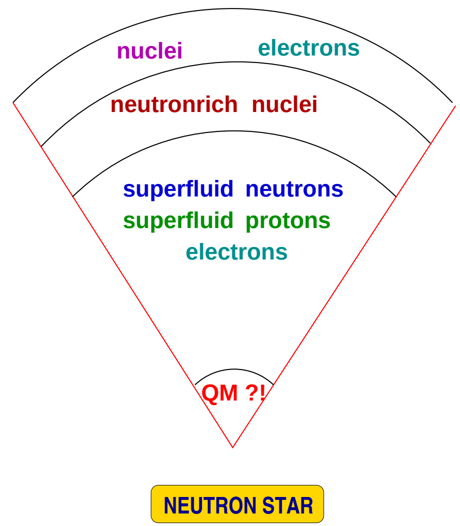
Quantum Chromodynamics

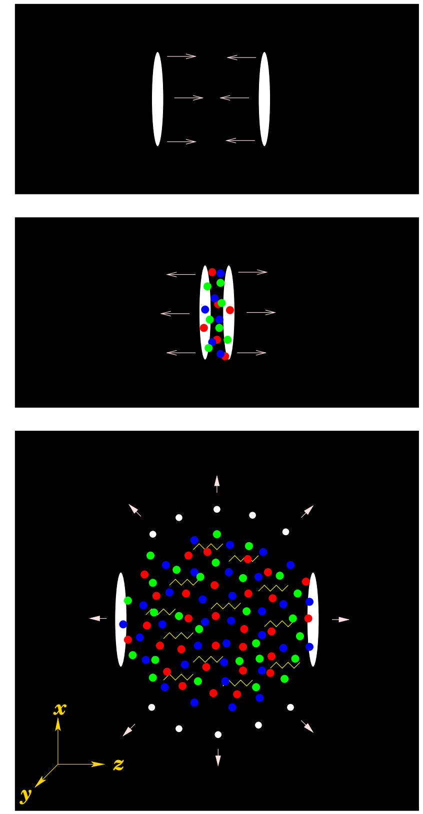


- Asymptotic Freedom of colour charge
  Politzer,Gross,Wilczek (1973)
- Confinement of colour charge
  Wilson (1974)

Finite T and μ







Heavy Ion Collision

Fireball

Rolf Hagedorn - Statistical bootstrap model (1960s)

- Hadron scattering shows *thermalization*
- As energy is increased newer states are excited
- Temperature goes to limiting value
- Entropy seems to diverge as spectrum grows exponentially

An important observation:

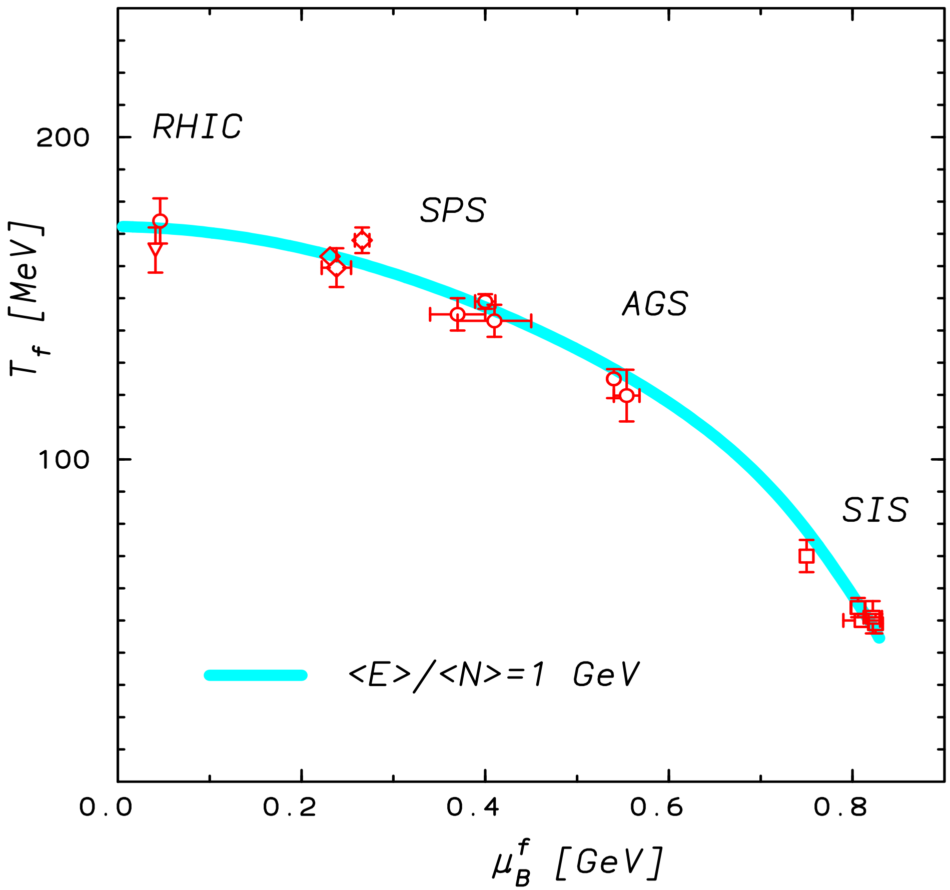
The information entropy of charged particles in hadron-hadron collisions at SPS as well as LHC is found to scale as $log(\sqrt{s})$.

V. Simak, M. Sumbera and I. Zborovsky, '88

P.A. Carruthers, M. Plumer, S. Raha and R.M. Weiner '88

S. Das, S.K. Ghosh, S. Raha and R. Ray '10

Thermalization



Thermal model:

$$Z(T, \mu_B, \mu_I, \mu_S) = \sum_i Z_i(T, \mu_B, \mu_I, \mu_S)$$

$$\Rightarrow n_i(T, \mu_B, \mu_I, \mu_S)$$

Braun-Munzinger, Redlich and Stachel: Particle production in heavy ion collisions

Pg. 491, 'Quark Gluon Plasma 3', R.C. Hwa and X.-N. Wang, ed., (World Scientific) (nucl-th/0304013)

QCD

$$\mathcal{L}_{QCD}^{E} = \frac{1}{4} F_{\mu\nu}^{a} F_{\mu\nu}^{a} + \sum_{f=1}^{N_f} \bar{q}_f \left(\gamma_\mu^E D_\mu + m_f - \mu_f \gamma_0 \right) q_f$$

where,

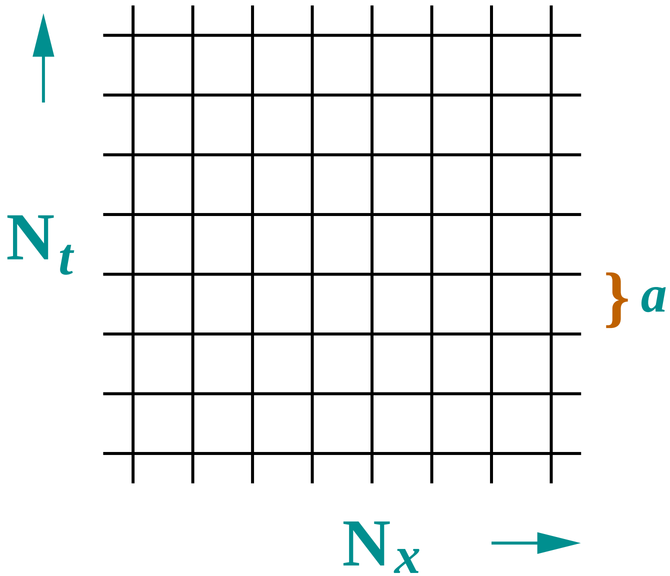
$$\begin{aligned} F_{\mu\nu}^{a} &= \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f^{abc} G_\mu^b G_\nu^c \\ D_\mu &= \partial_\mu - i g T^a G_\mu^a \qquad a = 1, 2, \ldots 8. \end{aligned}$$

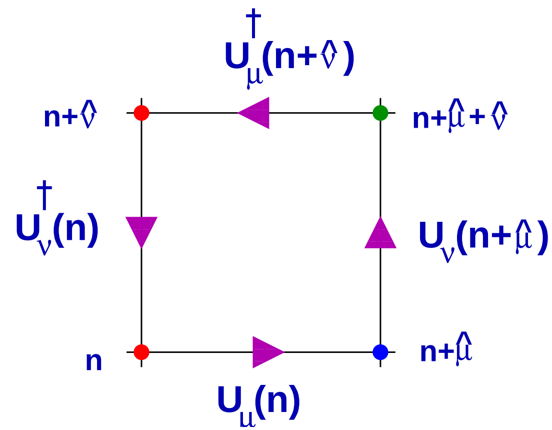
T^a are SU(3) group generators and f^{abc} are SU(3) structure constants

The QCD partition function is,

$$Z = \int DG_\nu^a Dq_f D\bar{q}_f \, \mathrm{e}^{-\int_0^\beta d\tau \int_{-\infty}^{\infty} d^3x \mathcal{L}_{QCD}^E}$$

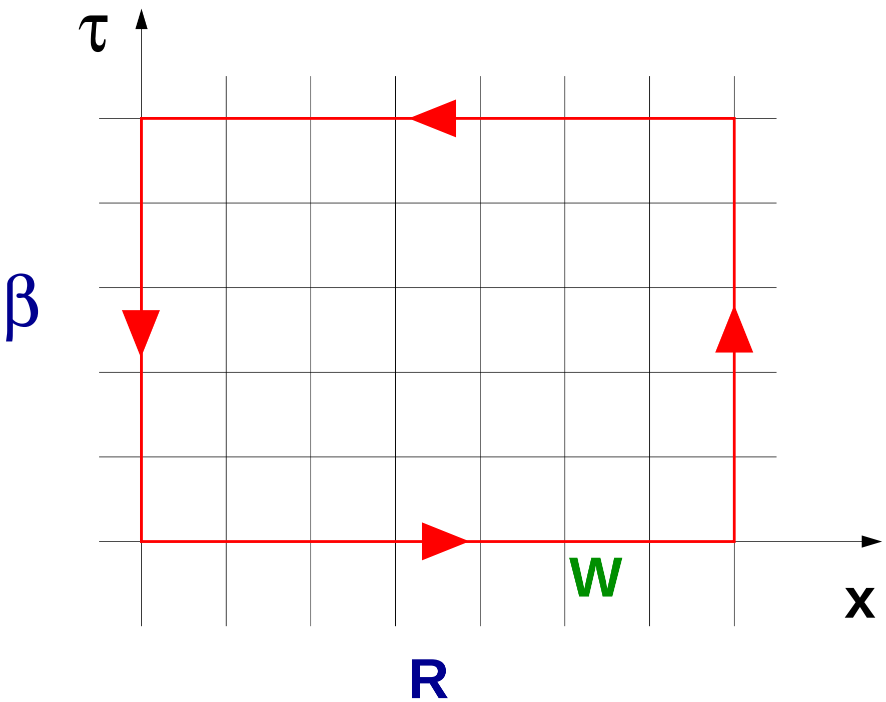
Lattice Formulation





- Quarks sit on the lattice points $q(\mathrm{n})$
- Gluons are on the links $U_\mu(\mathrm{n}) = \mathcal{P}\exp\left[ig\int_{\mathrm{n}}^{\mathrm{n}+\hat{\mu}a} \mathrm{d}y^\sigma\, G^a_\sigma(y)\, T^a\right]$
- $V = a^3(\mathrm{N}_x \times \mathrm{N}_y \times \mathrm{N}_z)$ $\qquad\qquad \beta = a\mathrm{N}_t$
- momentum cutoff $\simeq \frac{1}{a}$ $\qquad\qquad a \to 0 \Rightarrow$ Continuum physics

Confinement



$$\mathcal{C}\exp[-\beta\cdot \mathrm{V}(\mathrm{R})] = \langle \mathrm{W}(\mathrm{R},\beta)\rangle$$

Wilson Loop

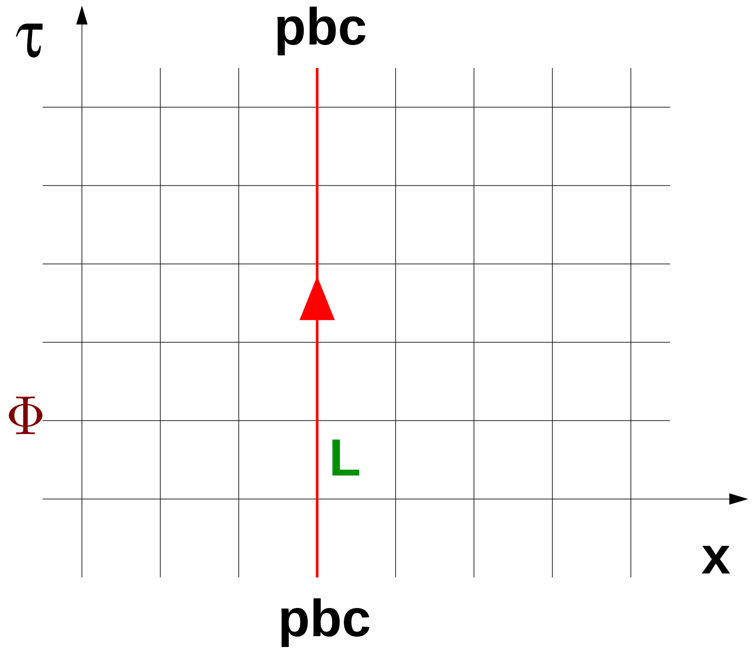
- Pair of q, $\bar{q}$ sitting a distance R apart, parallel transported in time τ.
- V(R) = $\begin{cases} -\frac{g^2}{3\pi \mathrm{R}} & \text{for weak coupling} \\ \frac{\mathrm{R}}{a^2}\ln(3g^2) & \text{for strong coupling} \end{cases}$

(De)Confinement

- Finite Temperature
  $\beta = 1/T$
- Free energy of vacuum
  $F_0 = -T \log Z$
- Free energy of a single quark
  $F_q - F_0 = -T \log \langle \mathrm{TrL}(\vec{x})/3 \rangle = -T \log \Phi$

  $\mathrm{L}(\vec{x}) = \mathcal{P}\exp\left[-\int_0^{1/T} \mathrm{d}\tau G_0(\vec{x},\tau)\right]$

  Wilson Line/Polyakov Loop



- $\Phi(\vec{x},\tau)$ = $\begin{cases} \neq 0 \Rightarrow F_q \text{ finite} \Rightarrow \text{deconfined} \\ = 0 \Rightarrow F_q \text{ infinite} \Rightarrow \text{confined} \end{cases}$
- Use Φ as OP for finite temperature phase transition.

Chiral Symmetry

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \sum_{f=u,d}\left[i\,\bar{q}_f\gamma^\mu D_\mu q_f - m_f\bar{q}_f q_f\right]$$

Symmetries: $SU(3)_c \otimes \underbrace{SU(2)_V \otimes SU(2)_A \otimes U(1)_B \otimes U(1)_A}_{Fermionic}$

- $U(1)_A$ broken by quantum anomalies.
- $SU(2)_V$ broken explicitly when flavour degeneracy is lifted

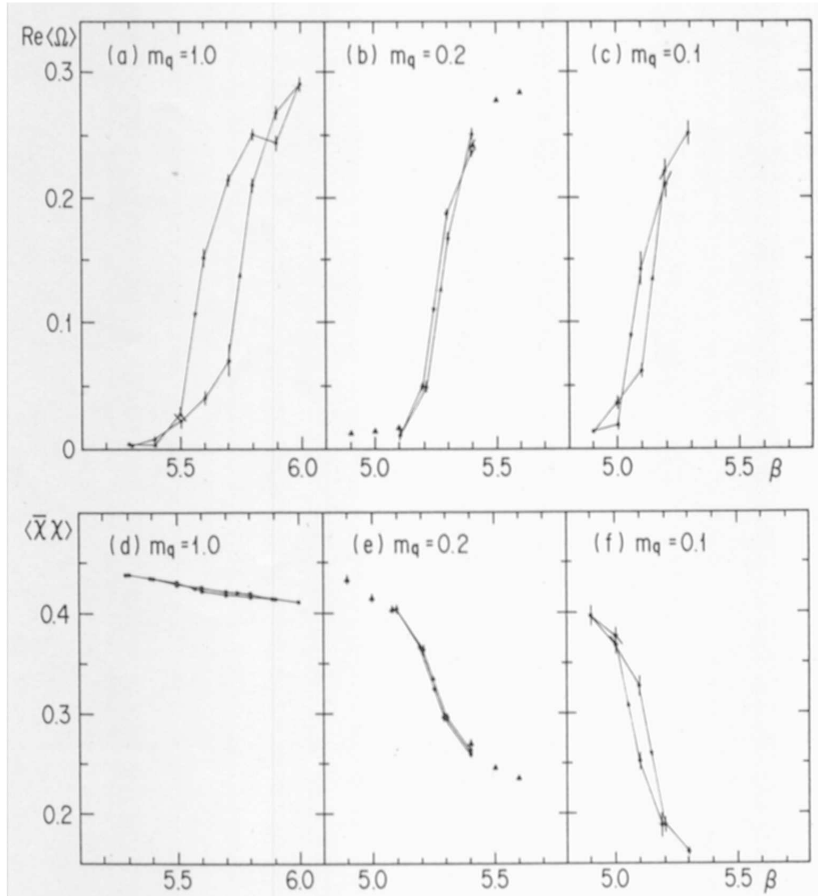
  e.g. proton and neutron mass splitting.
- $SU(2)_A$ broken explicitly for non-zero quark mass ....

  where are the chiral partners !!
- $SU(2)_A$ broken spontaneously; pions are the Goldstone Bosons

  $\rightarrow$ Measure is the chiral condensate $\langle\bar{q}q\rangle$.

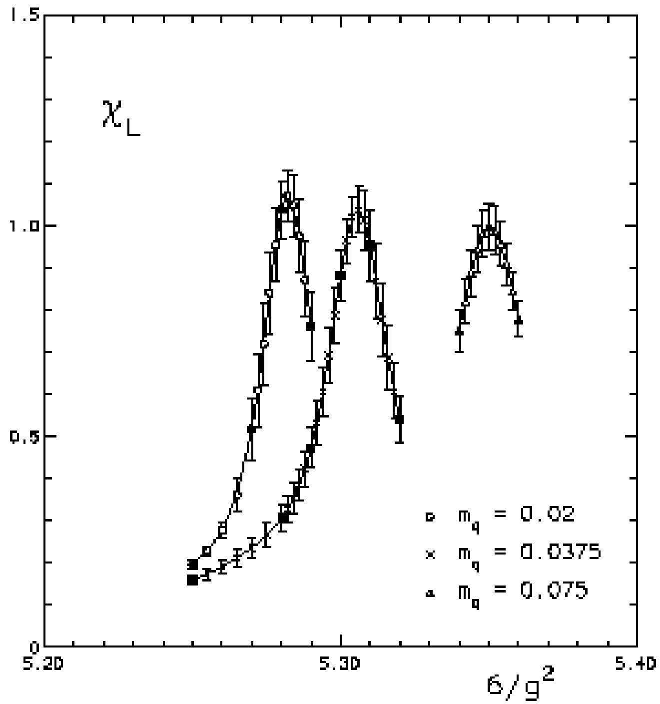
  $$\langle\bar{q}q\rangle = \begin{cases} \neq 0 \Rightarrow \text{symmetry broken} \\ = 0 \Rightarrow \text{symmetry restored} \end{cases}$$

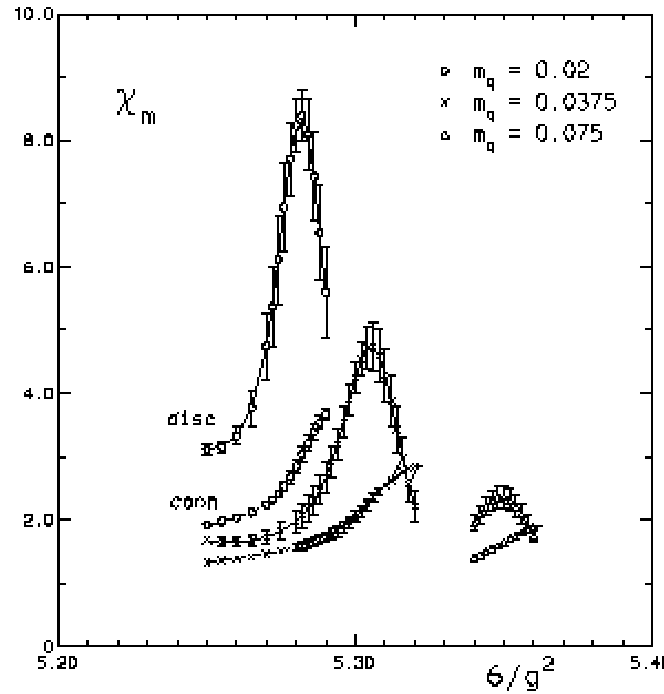
Order parameters



Variation of OPs Fukugita et.al. PRL 57 503 ’86

Coincidence





Variation of susceptibility Karsch et.al. PRD <u>50</u> 6954 '94

Recent estimates of transition temperature with 2 quark flavours:

$T_c = (171 \pm 4)$ MeV for Lattice Wilson fermions CP-PACS, PRD <u>63</u> 034502 '01

$T_c = (173 \pm 8)$ MeV for Lattice staggered fermions Bielefeld, NPB <u>605</u> 579 '02

Why bother about models

Get a workable intuitive understanding of strong interactions:

- N. Weiss formulated the one loop SU(N) effective potential both with and without fermions to understand confinement (1981)
- P.N. Meisinger and M.C. Ogilvie constructed a *covariant* formulation of Nambu-Jona-Lasinio model to understand the dependence of chiral transition in a confining background (1996)
- S. Digal, E. Laerman and H. Satz proposed Polyakov loop to be aligned by the chiral condensate (2000).
- R. Pisarski contructed a possible effective potential for the Polyakov loop by appealing to the center symmetry. (2000)
- P.N. Meisinger, T.R. Miller and M.C. Ogilvie constructed similar model which compare well with LQCD data above T_d. (2002)
- K. Fukushima incorporated an effective potential for the Polyakov loop in the Nambu-Jona-Lasinio model. (2003)

Why bother about models

Tread into territory difficult for QCD (away from $\mu_B = 0$)

Caution: First check out your worth with QCD

- C. Ratti, Thaler and W. Weise built up a Polyakov loop enhanced Nambu-Jona-Lasinio model for two light flavours highlighting the measurement of $P(T, \mu_B) - P(T, 0)$ (2006)
- The model was used by us to estimate and predict various susceptibilities contrasting them to results from QCD.
  S.K. Ghosh, T.K. Mukherjee, M.G. Mustafa and R. Ray (2006)
  S. Mukherjee, M.G. Mustafa and R. Ray (2007)
  S.K. Ghosh, T.K. Mukherjee, M.G. Mustafa and R. Ray (2008)
- The model was qualitatively established. There is still significant numerical mismatch.
- Results encouraging so try 2+1 flavor.
  Presentations by Paramita Deb and Anirban Lahiri

PNJL Model - 2 flavors

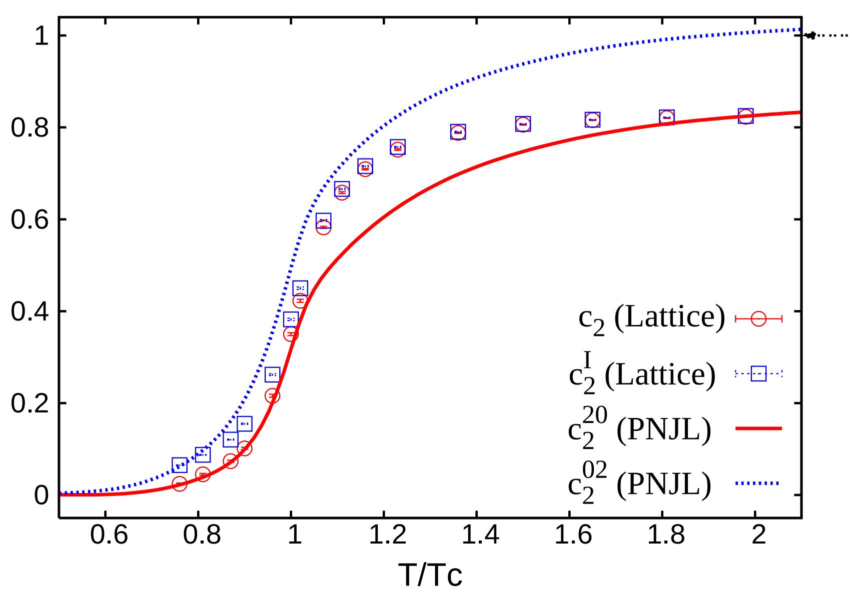
$$\begin{aligned}
\Omega &= \mathcal{U}\left(\Phi,\bar{\Phi},T\right) + 2G_1(\sigma_u^2+\sigma_d^2) + 4G_2\sigma_u\sigma_d \\
&- \sum_{f=u,d} 2\,T \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{ \ln\left[1+3\left(\Phi+\bar{\Phi}\mathrm{e}^{-(E_f-\mu_f)/T}\right)\mathrm{e}^{-(E_f-\mu_f)/T} + \mathrm{e}^{-3(E_f-\mu_f)/T}\right]\right. \\
&+ \left. \ln\left[1+3\left(\bar{\Phi}+\Phi\mathrm{e}^{-(E_f+\mu_f)/T}\right)\mathrm{e}^{-(E_f+\mu_f)/T} + \mathrm{e}^{-3(E_f+\mu_f)/T}\right]\right\} \\
&- \sum_{f=u,d} 6 \int \frac{\mathrm{d}^3p}{(2\pi)^3} E_f \theta\left(\Lambda^2 - \vec{p}^{\,2}\right)
\end{aligned}$$

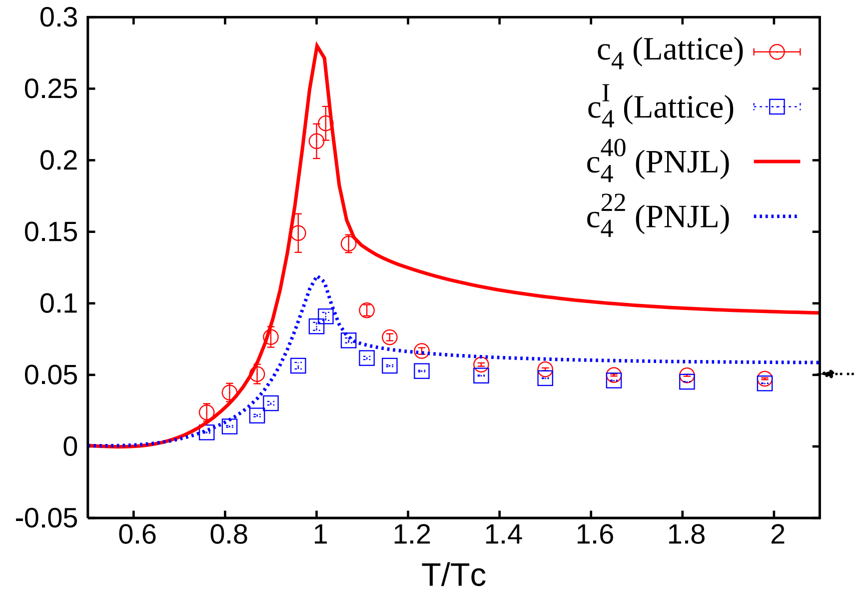
where,

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi},T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

$$+\kappa\ln[1-6\,\bar{\Phi}\Phi + 4\left(\Phi^3+\bar{\Phi}^3\right) - 3\left(\bar{\Phi}\Phi\right)^2]$$

Fluctuations of conserved charges



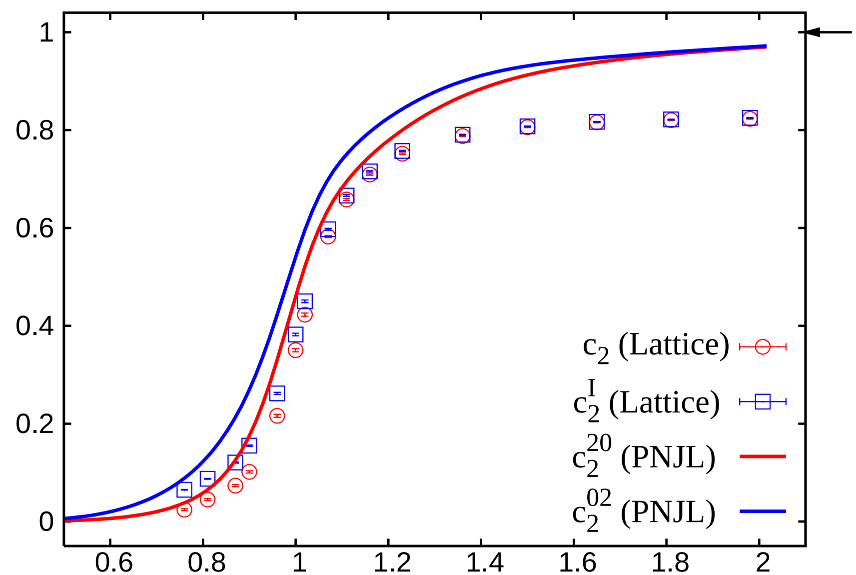


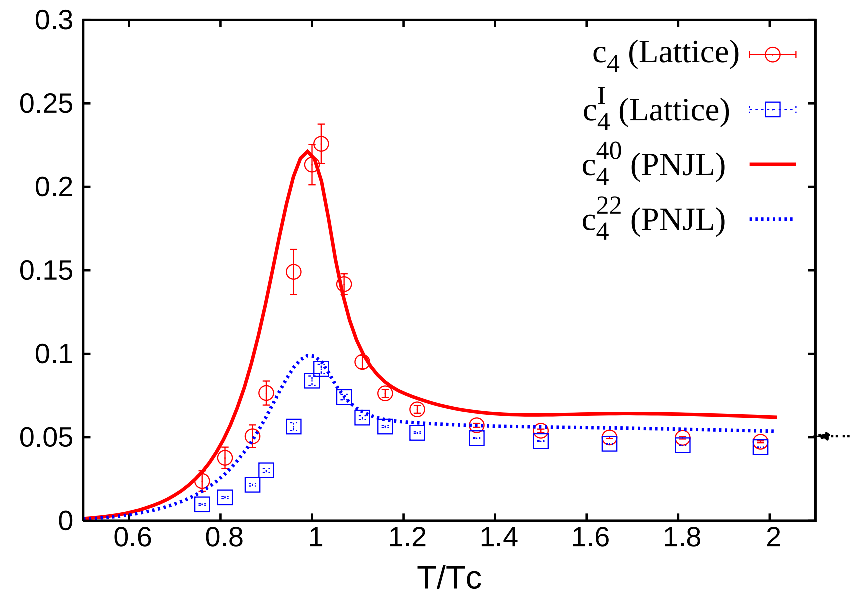
- Lattice: $c_2 \sim c_2^I \sim 80\%$ SB limit ; $c_4 \sim c_4^I \to$ SB limit
- PNJL: $c_2 \neq c_2^I$, $c_4 \neq c_4^I$

  c_2 and c_4 away from SB limit ; c_2^I and $c_4^I \to$ SB limit

PNJL model Mukherjee et.al. PRD '07 Lattice Bielefeld PRD 71 054508 '05

Fluctuation of conserved charges





- Lattice: $c_2 \sim c_2^I \sim 80\%$ SB limit ; $c_4 \sim c_4^I \to$ SB limit
- PNJL: $c_2 \sim c_2^I \to$ SB limit ; $c_4 \sim c_4^I \to$ SB limit
- At low temperatures PNJL coeff. greater than lattice.

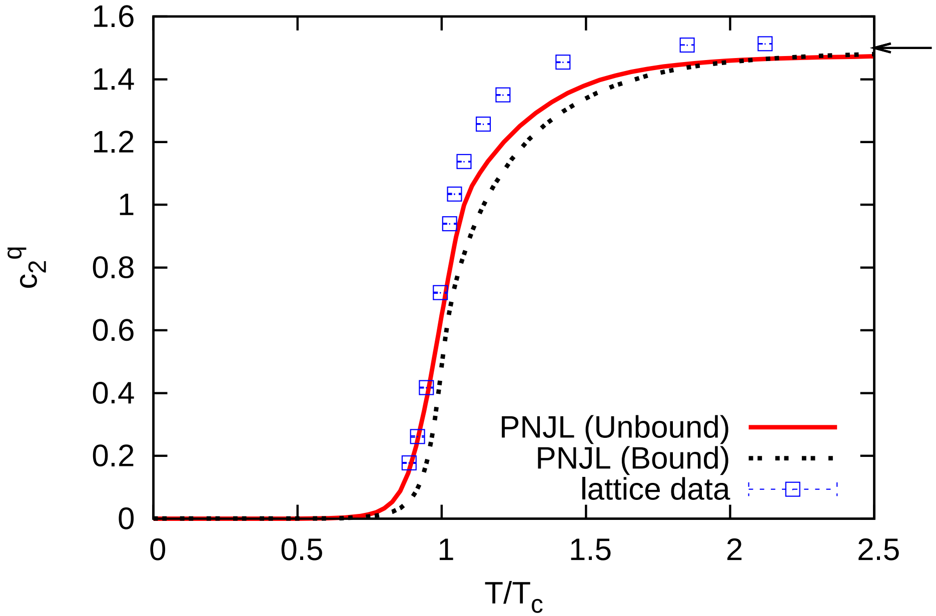
PNJL model Ghosh et.al. PRD '08

2+1 flavors

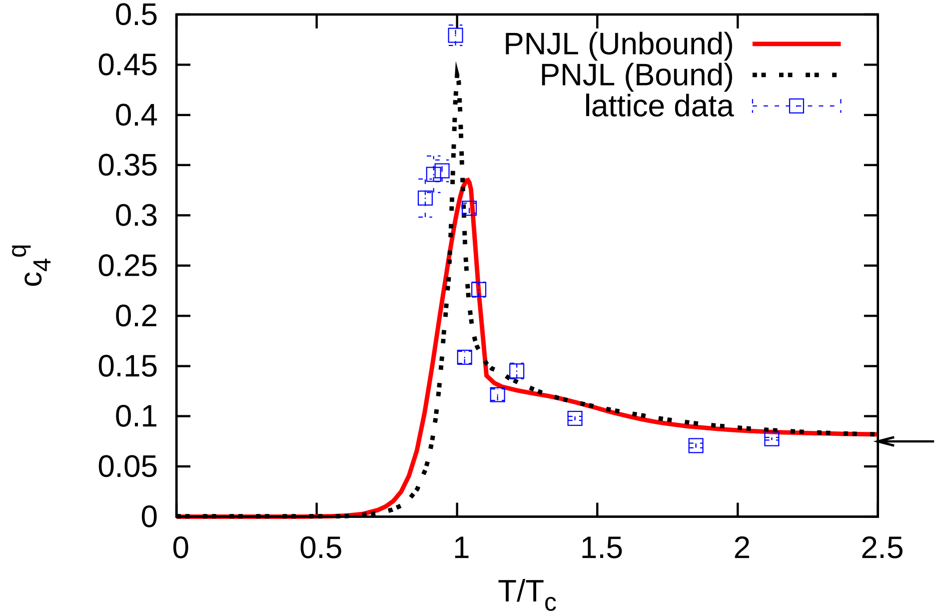
$$\begin{aligned}\Omega \quad &= \mathcal{U}'[\Phi,\bar{\Phi},T] + 2g_S\sum_{f=u,d,s}\sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s + 3\frac{g_1}{2}(\sum_{f=u,d,s}\sigma_f{}^2)^2 \\ &+3g_2\sum_{f=u,d,s}\sigma_f^4 - 6\sum_{f=u,d,s}\int_0^\Lambda \frac{d^3p}{(2\pi)^3}E_f\Theta(\Lambda-|\vec{p}|) \\ &-2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3(\Phi+\bar{\Phi}e^{-\frac{(E_f-\mu)}{T}})e^{-\frac{(E_f-\mu)}{T}}+e^{-3\frac{(E_f-\mu)}{T}}\right] \\ &-2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3}\ln\left[1+3(\bar{\Phi}+\Phi e^{-\frac{(E_f+\mu)}{T}})e^{-\frac{(E_f+\mu)}{T}}+e^{-3\frac{(E_f+\mu)}{T}}\right]\end{aligned}$$

where, g_S is the usual four-fermi interaction, g_D is the coupling for the ’t Hooft determinant and g_1 and g_2 are the 8q coupling constants needed to remove an infinite potential well close to the classical vacuum.

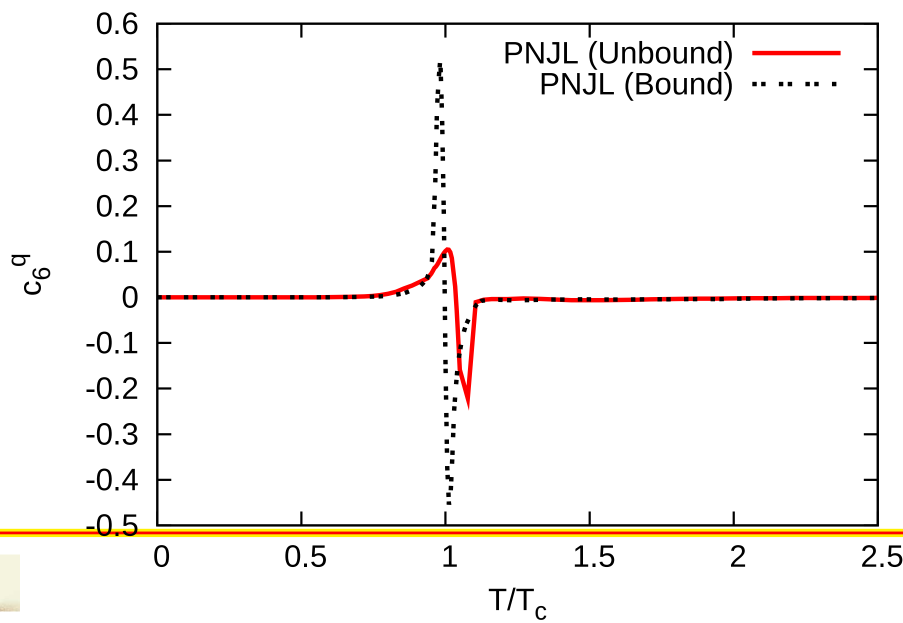
2+1 flavors



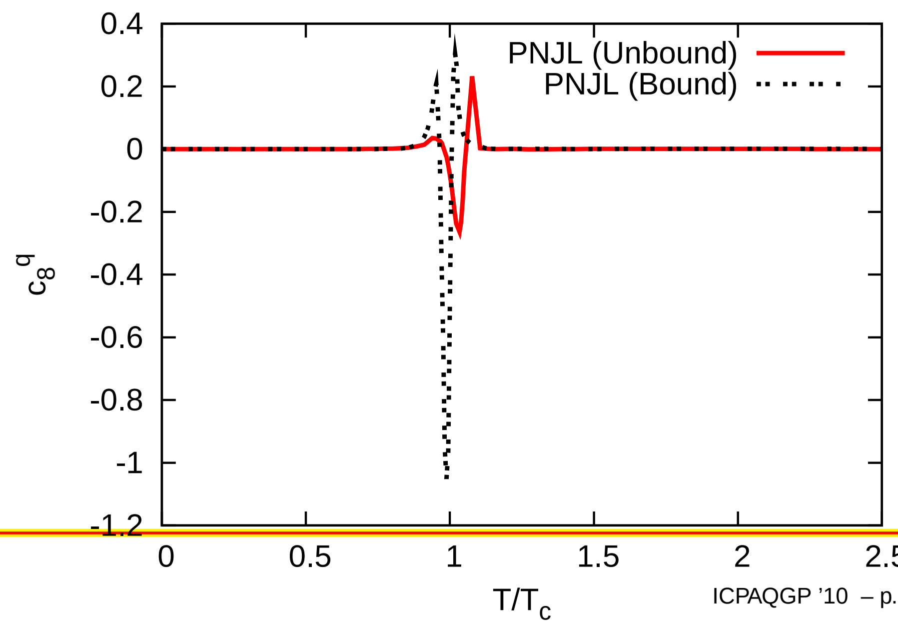
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
c_2^q
T/T_c
PNJL (Unbound)
PNJL (Bound)
lattice data



0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
2.5
c_4^q
T/T_c
PNJL (Unbound)
PNJL (Bound)
lattice data

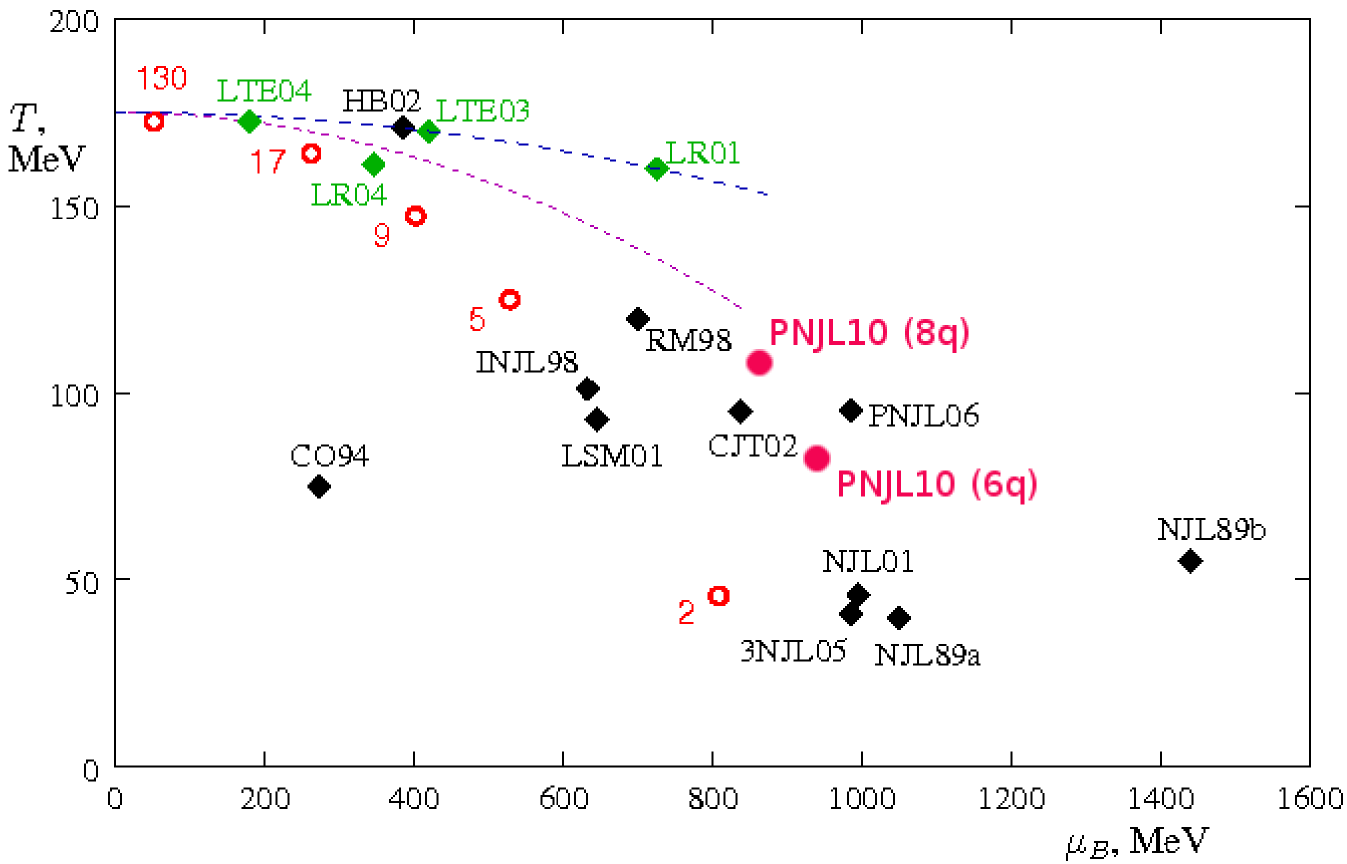


0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
c_6^q
T/T_c
PNJL (Unbound)
PNJL (Bound)



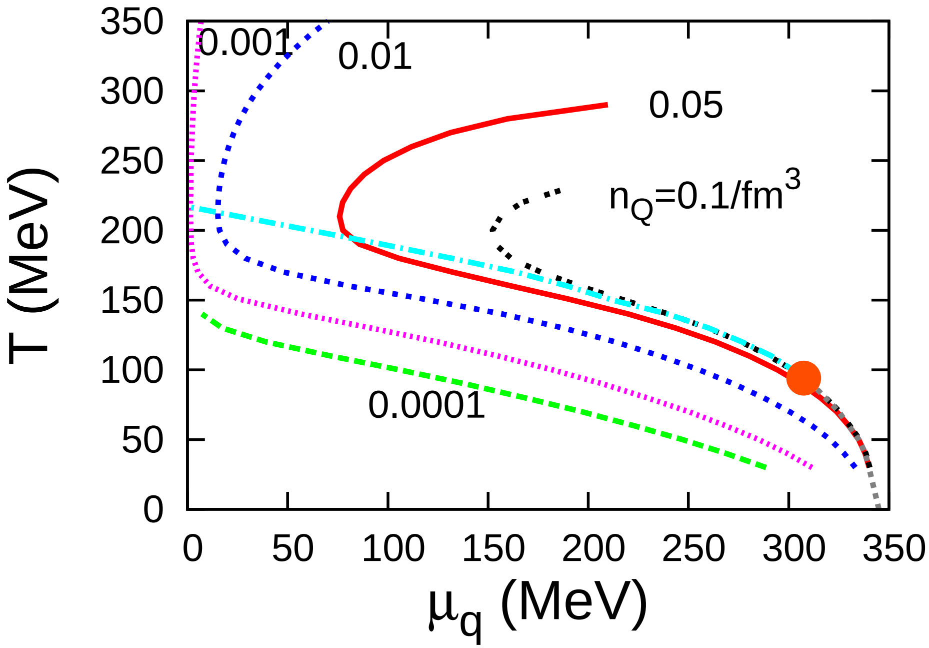
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.5
1
1.5
2
2.5
c_8^q
T/T_c
PNJL (Unbound)
PNJL (Bound)

Phase Diagram....



M. Stephanov (hep-lat:0701002)

Charge conservation....



Sarbani Mazumdar at satellite meeting TIFR, Mumbai

Outlook

- A number of models exist which in a certain region of validity describe aspects of strong interation physics very well.
- Various other models were not discussed DDQM (Plumer, Raha and Weiner), color dielectric (Neilson and Patkos) etc.
- A communication of ideas among the models would lead to better understanding of each other's strong points and weaknesses.
- Era of highly dependable models have come.

Enormous work has taken place in chiral models with background confinement. There are several presentation in the parallel sessions and posters (Deb, Haque, Lahiri, Moreira, Mukherjee, ...)