Models for Strong Interaction Physics

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Abstract

We review the developments in various models describing strong interaction physics. The models provide an intuitive way of understanding the complex phenomenon associated with strong interactions. Models also help us to delve into regions of couplings and other thermodynamic conditions of interest that are still out of reach of the first principle method - quantum chromodynamics. At the same to ascertain the merits of the models they should be contrasted to the results obtained from quantum chromodynamics at least in its region of validity, and to the available experimental data. Here we shall discuss about our progress in that direction.

Key words: Strong interaction, heavy-ion collision, phase diagram, fluctuations.
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1. Introduction

Understanding the bulk thermodynamic properties of strongly interacting matter has become a core issue in ultra-high energy heavy-ion collision experiments for the last few decades. The phase diagram of strong interaction physics is the holy grail that needs to be uncovered in the experimental and theoretical endeavors today. A wealth of knowledge exists on the relevant thermodynamic properties. Still it is far from being conclusive. The theory of strong interactions - Quantum Chromodynamics (QCD), predicts an extremely large coupling strength in the region of intermidiate temperatures and densities of the matter created by the current set of experiments. Thus usual perturbative formulation (pQCD) is supposed to be inadequate. Most often pQCD is useful in the initial state of the heavy-ion interactions and for the physics of hard particles produced in the interactions [1,2]. For the matter at very high temperatures also pQCD may be a good first principle method [3], though a more satisfactory description is obtained with the matter divided in hard and soft modes with an augmented approach - the Hard Thermal Loop (HTL) resummation schemes [4] and the HTL perturbation theory [5].

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To study the bulk thermodynamic properties of strongly interacting matter at intermidiate temperatures attained at the present heavy-ion accelarators, the only first principle framework seems to be the non-perturbative formulation of QCD with lattice regularization (LQCD) [6]. In this formulation the analytic calculation of the free energy of strong interactions is still a million dollar problem. One has to take resort to numerical computation, and that too with the best super computers put together. This is because, the free energy and all other thermodynamic observables are to be obtained by a direct implementation of the path integral method. With an enormous number of integration variables a huge computation cost has to be bourne. The continuum limit of LQCD, when obtained unambiguously, would finally lay down the answer to the physics of strong interactions. Present day LQCD computations have crossed a number of milestones in that direction, but yet to have reached the final goal.

Though the achievements of LQCD is commendable at zero densities, the path to finite densities is yet more troublesome. This is because of the inherent Monte Carlo integration scheme, where the statistical weight factor becomes complex. Given that no better alternative integration technique exits for such a huge multidimensional integration, one has to live with this problem and find regions in phase space where this problem is minimal. One such possibility is to use a significant overlap of configurations close to the transition region which will enable one to map out a phase boundary in the QCD phase diagram via multiparameter reweighting [7]. Another approach may be to expand the free energy as a Taylor series in the chemical potential with the coefficients evaluated at zero chemical potential [8]. Among other possibilities are the extraction of free energy for an imaginary chemical potential for which the Monte carlo weight factors are always real, and then analytically continuing the result to real chemical potentials [9]. This approach works in the region where chemical potential is less than the corresponding temperature in the phase diagram due to periodicity in the imaginary chemical potential direction [10]. Among other approches are the canonical ensemble method where the quark number is held fixed rather than the chemical potential for the commonly used grand canonical ensemble [11], and the density of states method [12] where the difficulty of complex weight factor is replaced by the difficulty in precisely determining the density of states. Note that all the above approaches are mainly concerned on how to lower or transfer the problem of complex weight factor in the Monte Carlo integration. An alternative integration technique would be more desirable to avoid this problem altogether.

2. Models for strong interaction

2.1. *Phenomenological models*

Thus most of the interesting information at the intermidiate chemical potentials we have today come from various QCD inspired models. Models in the field of strong interaction has a long history. In fact the birth of strong interaction happened through the satisfactory explanation of the hadron spectrum through the *constituent quark model* [13], following the success of the periodic table of hadrons - the *Eight-fold way* [14]. The success of the quark model led to the question as to whether these were physical and whether they can be observed. The experiments from 1966 to 1978 at the Stanford Linear Accelarator proved beyond doubt that the quarks are physical constituents of hadrons.

Finally a proper renormalizable theory - Quantum Chromodynamics - emerged to explain the physics of strong interactions [15]. The lack of observation of free quarks was understood as a physical confinement effect due to the sharp increase of running coupling of the interactions [16].

Working out any information in the strong coupling regime has however remained formidable even today. This gave rise to a number of interesting models for confinement physics. The most prominent are the *Bag model* and the *String model* [17]. In the Bag model the quarks are supposed to be free inside a confining boundary whereas in the String model the quarks are bound with a linear energy flux. Both these models gave a more or less satisfactory description of the hadron masses and spectrum. Even today the modern numerical event generators use the String fragmentation for hadronization of partons created in collider experiments [18]. Similarly, a modern avatar of the Bag model that has been very successful in describing data from LQCD is the quasi-particle model (QPM) [19]. Here the quarks as well as the transverse gluons form the soup of quasi-particles inside a bag, and lowest order pQCD effects are incorporated.

A parallel phenomenological approach to study multipartiparticle production was the statistical bootstrap model in which highly excited lumps of hadronic matter created were supposed to behave as particle resonances, leading to an exponential growth of the particle spectrum [20]. The particles and resonances may thermalize to form a *fireball*. With the spectrum growing exponentially the temperature of the fireball soon reaches a limiting value - the Hagedorn temperature. In the light of deconfinement physics this limiting temperature has today been seen to be close to the transition temperature of confined hadrons to deconfined quark gluon matter. The model is used nowadays to callibrate results in experimental and theoretical studies in strong interactions. [21]. Statistical models of strong interactions thus became fashionable. Studies of scaling of particle multiplicities became important. As the energies of collider experiments increased the scaling laws kept changing [22]. Later entropy scaling was proposed to give the ultimate scaling law for multiparticle production [23]. In terms of chaotic and coherent sources the final picture was obtained for center of mass energy ranging from 22 GeV to 900 GeV per incident particle [24]. With the current LHC data at center of mass energy 2.3 TeV per incident particle we find this scaling to still hold true [25]. Today the most popular statistical model is the thermal model in which all known particles and resonances are identified (directly or indirectly) in the experiments and are then put into corresponding Bose or Fermi statistics to get the best set of unique thermodynamic parameters. Surprisingly enough the model does indeed give a satisfactory description of particle production [26].

2.2. *Symmetries and Mean field models*

Another set of models that has become quite important in understanding the equation of state of strongly interacting matter are mean field models. These employ field theoretic techniques to study the dynamics of low momentum modes with the effect of high momentum modes put inside the various couplings and parameters of the models. The basic field variable for the confinement phenomenon was the Wilson loop at zero temperature [16] and the Wilson line or Polyakov loop [27] at finite temperature [28].

That the strong interaction vacuum can act as a color dielectric was seen in effective models of the Wilson loop [29]. The value of the dielectric field in a region would decide whether the region is in hadronic vacuum or in deconfined vacuum. In a model that uses the dielectric model to determine the vacuum and quasi-particle picture of the corresponding excitations, we explored the dynamics of the fireball created in collider experiments [30]. The dynamics has the potential to generate a hot spot in the later stages with temperatures far exceeding that of the orginal fireball.

Considering the global center symmetry in a non-abelian SU(N) theory, and its spontaneous breaking at high temperatures, the Polyakov loop is a suitable order parameter. Deconfinement of hadronic matter could thus be described in terms of the change of the Polyakov order parameter with temperature in QCD. Efforts in LQCD were on to identify this symmetry breaking deconfinement transition. Alongside, effective Polyakov loop models were built up with terms consistent with the Z(3) center symmetry [31]. Many of these models are parametrized satisfactorily with a fit to the available LQCD data. Applications of these models in pure gauge SU(3) theory may be found in the literature [32].

In QCD there is a bigger set of global symmetries associated with the Dirac part of the Lagrangian for quarks. These are the quark number, isospin and chiral symmetries. Now isospin and chiral symmetry being respectively vector and pseudo-vector in nature, require the existence of corresponding multiplets. In the lightest quark sector we can find the *up* and *down* quarks to be the vector partner of each other and their corresponding left-handed and right-handed components are the chiral partners. For the lowest lying states in the hadronic sector one can consider the proton and neutron to be the vector partners of each other. However no particles exist to compliment their chiral symmetry. This global symmetry is thus expected to be spontaneously broken. Note that spontaneous symmetry breaking is essentially a property of the ground state and hence the lowest lying states manifest some of that effect. Higher excited states may have the symmetry restored [33]. Surely enough, the chiral symmetry led to various chiral Lagrangians through a systematic study of chiral currents and chiral perturbation theory [34]. The popular sigma model [35] and the Nambu-Jona-Lasinio (NJL) model [36] are among the oldest chiral models in use. The modern treatment of the respective models can be found in Ref. [37] and [38]. Some time back, using the linear sigma model we showed [39] that a clean signal of formation of *disoriented chiral condensates* [40] is indeed possible in heavy-ion collision experiments. Also on the Lattice we explored the behaviour of the chiral order parameter at non-zero temperature and chemical potential and discussed its implication on the QCD phase diagram [41].

Thus far the two most important aspects of QCD, namely confinement and chiral symmetry breaking was being studied separately in the effective models. On the other hand simulations of QCD on the lattice showed a surprising coincidence [42] of the two phenomenon at almost the same temperature [43]. Work begun to tie up these two phenomenon together in effective models. The interplay of confinement and chiral symmetry breaking, essentially the simultaneous onset of these two phenomenon were studied [44]. Thereafter, various dynamical models suitable for QCD thermodynamics emerged. Most notable are those which couple the Polyakov loop model to either the NJL model (called PNJL models) [45–48], or the sigma model with quarks (called PQM/PLSM models) [49].

The main motivation for working with dynamical models is to be able to have a real time description of the evolution of the early universe, the core of supermassive stars and that of the particle collision experiments. The evolution may or may not be in equilibrium. Such a dynamical description is out of scope of any first principle calculation of QCD. At the same time the model should also reproduce correct results in those limits for which systematic QCD results are available. Therefore over the last few years there has been a vast increase in the activity of recursively formulating and parametrizing models with newer QCD physics inputs and LQCD data. In the rest of the talk we shall focus on the studies of QCD thermodynamics based on the PNJL model.

2.3. *PNJL model*

The first job in the studies of the dynamical effective models is to obtain the equation of state (EOS) and various susceptibilities corresponding to conserved charges. Susceptibilities in general, are related to fluctuations via the fluctuation-dissipation theorem. The fluctuations of various conserved quantities like baryon number, electric charge etc. would be different in the hadronic and deconfined phases and act as a signal of the deconfining transition in heavy-ion reactions [50]. Measurements of these fluctuations have taken a central place in the heavy-ion collisions [51]. Computations on the Lattice have given us many of these susceptibilities at zero chemical potential [52,53]. Thus one can extract these quantities from the PNJL model and compare with LQCD results.

There are other quantities of interest that one can obtain in PNJL model. Among these are the specific heat, speed of sound and conformal measure. The specific heat C_V, is related to the event-by-event temperature fluctuations [54], and mean transverse momentum fluctuations [55] in heavy-ion reactions. The speed of sound (basically its square, v_s^2) determines the flow properties in heavy-ion reactions [56–59]. Using the proper hydrodynamic equations including the speed of sound it is possible to analyze the multiplicity distribution of the produced particles in collision experiments [60]. Finally the conformal measure $\mathcal{C} = \Delta/\epsilon$, where $\Delta = \epsilon - 3P$ is the interaction measure and ϵ and P are respectively the energy density and pressure of strongly interacting matter. As has been pointed out in [61,62], the conformal measure seems to be emerging as an important measure to draw similarities between long distance physics of QCD and conformal field theory, with results coming from both the areas of AdS/CFT correspondence [63], RHIC data [64] and Lattice computations [65].

The first ever study of these fluctuations in a dynamical model framework with a direct contrast to the LQCD data was done using the PNJL model in [47,48] with two quark flavors. The results were highly encouraging and led to further exploration in terms of extending the model by taking into account various possible interactions that were left out in the original model. In our attempt to look into the flavor mixing effects we found that the correlation between the up and down quark flavors i.e. the off-diagonal susceptibility is exceptionally large at high temperatures [66]. This was unexpected from pQCD as well as from LQCD results. It was rectified by extending the Polyakov loop potential with the corresponding Haar measure [67]. Without the Haar measure the Polyakov loop was approaching to an unphysical value leading to the unphysical behaviour of the off-diagonal susceptibility. Improvements in the NJL sector have been mainly concentrated in the regularization schemes [68] and extensions in terms of adding different kinds of multi-quark interactions e.g. the di-quark interaction [69], eight-quark interactions [70] etc. and extensions beyond mean field [71].

Various studies of chiral perturbation theory, strong coupling lattice gauge theory and chiral meson models indicate [72–76] that at low temperatures there is a possibility of first order phase transition for a large baryon chemical potential μ_{B_c}. This μ_{B_c} is supposed to decrease with increasing temperature. Thus there is a first order phase transition line starting from ($T = 0$, $\mu_B = \mu_{B_c}$) on the μ_B axis in the (T,μ_B) phase diagram which steadily bends towards the ($T = T_c$, $\mu_B = 0$) point and may actually terminate at a critical end point (CEP) characterized by ($T = T_E$, $\mu_B = \mu_{B_E}$), which can be detected via enhanced critical fluctuations in heavy-ion reactions [77]. The location of this CEP has become a topic of major importance in effective model studies (see e.g. Ref.[75,78]). As mentioned earlier that for $\mu_B \neq 0$ LQCD has a problem of complex weight functions in the Monte Carlo integrations, which hinders usual importance sampling techniques. However, the CEP was located for the physical [79] and for somewhat larger [80] quark masses using the reweighting technique of [81], and for Taylor expansion method in [82]. For nonzero isospin chemical potential (μ_I) models and effective theories [83] find an interesting array of possible phases. The most important phenomenon that is supposed to happen is a transition to the pion condensed phase close to $\mu_I \sim m_\pi$. This has also been supported by Lattice simulations [84], which does not suffer from the complex determinant problem for $\mu_I \neq 0$ and $\mu_B = 0$. The LQCD determination of the CEP is quite different from that obtained from the NJL or from the basic PNJL model. Addition of eight quark coupling was found to give much better agreement with LQCD results [70]. The phase diagram in $T - \mu_I$ plane is also found to be in good agreement. Effects of color neutrality on the phase diagram has also been reported [85]. The existence of the exotic phases of color superconductor [86] and color-flavor-locked phases [87] have also been investigated. Even the phase of quarkyonic matter [88] seems quite natural in the PNJL model framework [89].

Among other quantities considered within the two flavor PNJL model are the mesonic correlations and scattering lengths [90]. Similar studies on meson masses and correlations for two flavor PQM model can be found in [91] and for 2+1 flavor PNJL model in [92].

The most commonly used 2+1 flavor NJL sector of the PNJL model is with a four-quark and six-quark effective potential [93,94,92]. However such an effective potential is found to be unbound which can be cured by introducing eight quark interaction terms [95]. We developed the 2+1 flavor NJL (and hence PNJL) model with bound effective potential within a three-momentum cutoff scheme to make it suitable for studying finite temperature and chemical potential properties [96]. This extra interaction brings the CEP somewhat closer to the value predicted from LQCD. For 2+1 flavors, various fluctuations have been measured in LQCD [97,98] as well as in PNJL model both with the usual unbound effective potential [99,100] and with the bound effective potential [101]. Similar calculations have been carried out in Polyakov loop coupled quark-meson (PQM) model [102] and its renormalization group improved version [103]. Recently, we also studied the correlators of the baryon-strange, baryon-charge and charge-strange quantum numbers both for bound and unbound effective potentials and the LQCD data [104]. In all the above cases PNJL results have reasonable qualitative agreement with Lattice data however the quantitative diviation is quite a lot especially in the fluctuations or correlations related to the strange sector. Could this be because of some physics missing in the model or could this be the systematic uncertainty of the LQCD data only time will tell.

3. Conclusion

In this talk we reviewed the status of various models for strongly interacting matter that developed over the last five decades. Recent efforts is evidently looking promising as the bits and pieces of the big puzzle is steadily being collected. The loose ends are being tied up with proper improvements so as to be consistent with the pQCD and LQCD results in their respective region of validity. Though one may argue that there is no guarantee that models would be equally good in the other regions, but still to bridge the gap between pQCD and LQCD, models are the best bet for now. One area where a lot of work has to be done to realize the full potential of the models, is to have a proper dynamical description of the system in real time.

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