

# Extended Nambu–Jona-Lasinio Model with covariant regularization

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# Outline

## 1 Introduction and formalism

- The Nambu–Jona-Lasinio Model
- The model Lagrangian
- Polyakov Loop
- 3D Regularization
- Pauli-Villars regularization

## 2 Results

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- Results using  $U^{II}$  in the PNJLH model
- Results using  $U^{III}$  in the PNJLH model
- Results using  $U^{IV}$  in the PNJLH8q model
- Some considerations on divergences with temperature

## 3 Conclusions

- Final slide
- References

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- **Quark condensates** as order parameter
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- Local and non renormalizable

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## ■ Nambu–Jona-Lasinio (4 q)

$$\mathcal{L}_{NJL} = \frac{G}{2} \left[ (\bar{q} \lambda_a q)^2 + (\bar{q} \not{v} \gamma_5 \lambda_a q)^2 \right]$$

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- 't Hooft determinant (6 q)

$$\mathcal{L}_H = K (\det \bar{q} P_L q + \det \bar{q} P_R q)$$

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- $\mathcal{L}_H = K (\det \bar{q} P_L q + \det \bar{q} P_R q)$
- **Eight quark interaction term**

$$\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}$$

$$\mathcal{L}_{8q}^{(1)} = 8g_1 [(\bar{q}_i P_R q_m) (\bar{q}_m P_L q_i)]^2$$

$$\mathcal{L}_{8q}^{(2)} = 16g_2 (\bar{q}_i P_R q_m) (\bar{q}_m P_L q_j) (\bar{q}_j P_R q_k) (\bar{q}_k P_L q_i)$$

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**OZI** violation in  $\mathcal{L}_H$  and  $\mathcal{L}_{8q}^{(1)}$ .

# Thermodynamical potential and stationary phase conditions NJLH8q case

For details on how to get here from the previous slide see: Phys. Rev. D 81, 116005 (2010); 0812.1532 [hep-ph]



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$$\begin{aligned} \Omega (M_f, T, \mu, \phi, \bar{\phi}) &= \\ &= \frac{1}{16} \left( 4Gh_f^2 + \kappa h_u h_d h_s + \frac{3g_1}{2} (h_f^2)^2 + 3g_2 h_f^4 \right) \Big|_0^{M_f} \\ &+ \frac{N_c}{8\pi^2} \sum_{f=u,d,s} \left( J_{-1}(M_f^2, T, \mu) + C(T, \mu) \right) \end{aligned}$$

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$$\begin{cases} m_u - M_u = Gh_u + \frac{\kappa}{16} h_d h_s + \frac{g_1}{4} h_u h_f^2 + \frac{g_2}{2} h_u^3 \\ m_d - M_d = Gh_d + \frac{\kappa}{16} h_u h_s + \frac{g_1}{4} h_d h_f^2 + \frac{g_2}{2} h_d^3 \\ m_s - M_s = Gh_s + \frac{\kappa}{16} h_u h_d + \frac{g_1}{4} h_s h_f^2 + \frac{g_2}{2} h_s^3 \end{cases}$$

# Inclusion of the Polyakov loop.

Introduce homogeneous background  $A_4$  gluonic field

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + iA^\mu, \quad A^\mu = \delta_0^\mu g A_a^0 \frac{\lambda^a}{2}, \quad L = \mathcal{P} e^{\int_0^\beta dx_4 iA_4},$$

$$\phi = \frac{1}{N_c} \text{Tr} L, \quad \bar{\phi} = \frac{1}{N_c} \text{Tr} L^\dagger$$

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- approximate **order parameter** (exact in the quenched limit) for (de)/confinement ( $\phi = 0 \leftrightarrow$  confined)
- enters the action as an **imaginary  $\mu$**

$$n_q(E_p, \mu, T) = \frac{1}{1 + e^{(E_p - \mu)/T}} \rightarrow \tilde{n}_q(E_p, \mu, T, \phi, \bar{\phi}) \equiv \frac{1}{N_c} \sum_{i=1}^{N_c} n_q(E_p, \mu + \imath (A_4)_{ii}, T)$$

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- Extra term, the **Polyakov potential**:  $\mathcal{U}(\phi, \bar{\phi}, T)$

# Thermodynamical potential with Polyakov loop

For details see: 1008.0569[hep-ph]

$$\begin{aligned}
 \Omega(M_f, T, \mu, \phi, \bar{\phi}) &= \\
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 \end{aligned}$$

# Polyakov potentials

Polyakov potential	Parameters																
$\frac{\mathcal{U}^I}{T^4} = -\frac{b_2(T)}{2}\bar{\phi}\phi - \frac{b_3}{6}(\phi^3 + \bar{\phi}^3) + \frac{b_4}{4}(\bar{\phi}\phi)^2$ $b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$	<table> <tr> <td><math>a_0</math></td> <td><math>a_1</math></td> <td><math>a_2</math></td> <td><math>a_3</math></td> </tr> <tr> <td>6.75</td> <td>-1.95</td> <td>2.625</td> <td>7.44</td> </tr> <tr> <td><math>b_3</math></td> <td><math>b_4</math></td> <td></td> <td></td> </tr> <tr> <td>0.75</td> <td>7.5</td> <td></td> <td></td> </tr> </table>	$a_0$	$a_1$	$a_2$	$a_3$	6.75	-1.95	2.625	7.44	$b_3$	$b_4$			0.75	7.5		
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$\mathcal{U}^I$  from C. Ratti, M.A. Thaler, and W. Weise, Phys. Rev. D 73, 014019 (2006)

$\mathcal{U}^{II}$  from S. Roessner, C. Ratti, W. Weise, Phys. Rev. D 75, 034007 (2007)

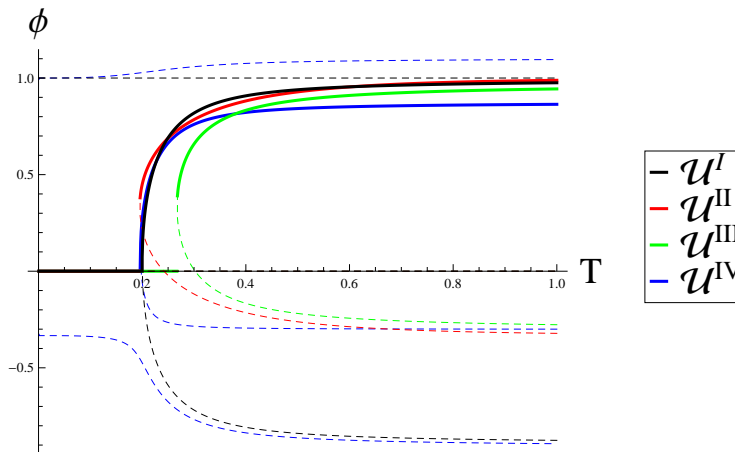
$\mathcal{U}^{III}$  from K. Fukushima, J. Phys. G 35, 104020 (2008); arXiv:0806.0292 [hep-ph]

$\mathcal{U}^{IV}$  from A. Bhattacharyya, P. Deb, S. K. Ghosh, R. Ray, arXiv:1003.3337v1 [hep-ph]





# Temperature dependence of the minima of the considered Polyakov potentials



Solutions of  $\frac{\partial}{\partial \phi} \mathcal{U} = 0$

$$3D: \hat{\rho}^{3D} = \Theta(\Lambda - |\vec{p}_E|)$$

- $\int_0^\infty d|\vec{p}_E| \hat{\rho}^{3D} f(|\vec{p}_E|) = \int_0^\Lambda d|\vec{p}_E| \hat{\rho}^{3D} f(|\vec{p}_E|)$

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- **Vacuum part**:
 
$$\begin{aligned}
 J_{-1}^{vac}(M^2) &= -16\pi^2 \int_0^{M^2} d\sigma^2 \int \frac{d^4 p_E}{(2\pi)^4} \hat{\rho}^{3D} \frac{1}{p_4^2 + \vec{p}_E^2 + \sigma^2} \\
 &= \Lambda \left( 2\Lambda^3 - \sqrt{M^2 + \Lambda^2} (M^2 + \Lambda^2) \right) + M^4 \text{ArcSinh} \frac{\Lambda}{M}
 \end{aligned}$$

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- $M$  independent:**

$$C(T, \mu) = \int_0^\Lambda d|\vec{p}_E| 8|\vec{p}_E|^2 T \ln \left( \left( 1 + e^{\frac{|\vec{p}_E| + \mu}{T}} \right) \left( 1 + e^{-\frac{|\vec{p}_E| - \mu}{T}} \right) \right)$$

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- Polyakov loop inclusion:**  $n_q(M, P, T, \mu) \rightarrow \tilde{n}_q(M, P, T, \mu, \phi, \bar{\phi}), \dots$

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$$J_{-1}(M^2, T, \mu) = J_{-1}^{vac}(M^2) + J_{-1}^{med}(M^2, T, \mu)$$

- Vacuum part:**

$$\begin{aligned} J_{-1}^{vac}(M^2) &= -16\pi^2 \int_0^{M^2} d\sigma^2 \int \frac{d^4 p_E}{(2\pi)^4} \hat{\rho}^{3D} \frac{1}{p_4^2 + \vec{p}_E^2 + \sigma^2} \\ &= \Lambda \left( 2\Lambda^3 - \sqrt{M^2 + \Lambda^2} (M^2 + \Lambda^2) \right) + M^4 \text{ArcSinh} \frac{\Lambda}{M} \end{aligned}$$

- Medium part:**  $J_{-1}^{med}(M^2, T, \mu) = - \int_0^{M^2} d\sigma^2 \int_0^\Lambda d|\vec{p}_E| \frac{4|\vec{p}_E|}{\sqrt{\sigma^2 + |\vec{p}_E|^2}} (-n_q - n_{\bar{q}})$

- $M$  independent:**

$$C(T, \mu) = \int_0^\Lambda d|\vec{p}_E| 8|\vec{p}_E|^2 T \ln \left( \left( 1 + e^{\frac{|\vec{p}_E| + \mu}{T}} \right) \left( 1 + e^{-\frac{|\vec{p}_E| - \mu}{T}} \right) \right)$$

- Polyakov loop inclusion:**  $n_q(M, P, T, \mu) \rightarrow \tilde{n}_q(M, P, T, \mu, \phi, \bar{\phi}), \dots$

- $\hat{\rho}^{3D, \infty}$ : remove cutoff just in medium part



$$\text{Pauli-Villars: } \hat{\rho}_{\Lambda \vec{p}_E} = 1 - \left(1 - \Lambda^2 \partial_{\vec{p}_E^2}\right) \exp\left(\Lambda^2 \partial_{\vec{p}_E^2}\right)$$

■ PV regulator:  $\hat{\rho}_{\Lambda \vec{p}_E} f(|\vec{p}_E|^2) = f(|\vec{p}_E|^2) - f(|\vec{p}_E|^2 + \Lambda^2) + \Lambda^2 \frac{\partial}{\partial |\vec{p}_E|^2} f(|\vec{p}_E|^2 + \Lambda^2)$

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- **Polyakov loop inclusion:**  $n_q(M, p, T, \mu) \rightarrow \tilde{n}_q(M, p, T, \mu, \phi, \bar{\phi}), \dots, n_q(0, p, T, \mu)$   
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- $\hat{\rho}^{PV, \infty}$ : remove cutoff ( $\hat{\rho} = 1$ ) just in medium part

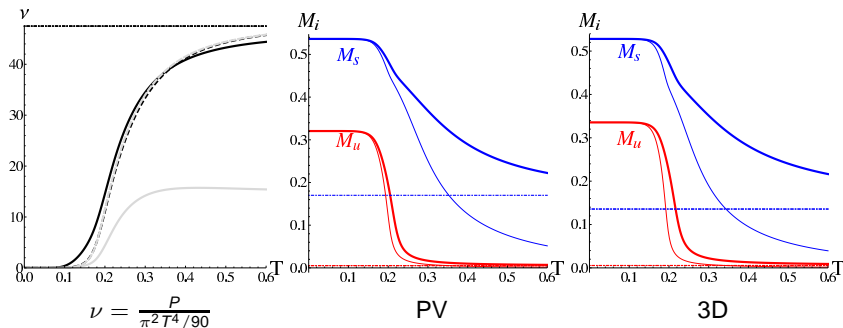
# Parameters for the quark interaction part

Sets	$m_u$ (MeV)	$m_s$ (MeV)	$\Lambda$ (MeV)	$G$ (GeV <sup>-2</sup> )	$\kappa$ (GeV <sup>-5</sup> )	$g_1$ (GeV <sup>-8</sup> )	$g_2$ (GeV <sup>-8</sup> )
I	5.3	170	920	8.89	-687	0	0
II	5.5	135.7	631.4	9.21	-740.6	0	0
III	5.9	186	851	7.03	-1001	1000	-47
IV	5.5	183.468	637.720	7.165	-720.245	2193	-589

- Set I: A.A. Osipov, A.H. Blin, B. Hiller; 0410148 [hep-ph]
- Set II: T. Hatsuda, T. Kunihiro, Phys. Rep. 247, 221 (1994)
- Set III: B. Hiller, J. Moreira, A.A. Osipov, A.H. Blin, Phys. Rev. D 81, 116005 (2010); 0812.1532 [hep-ph]
- Set IV: A. Bhattacharyya, P. Deb, S. K. Ghosh, R. Ray; 1003.3337v1 [hep-ph]



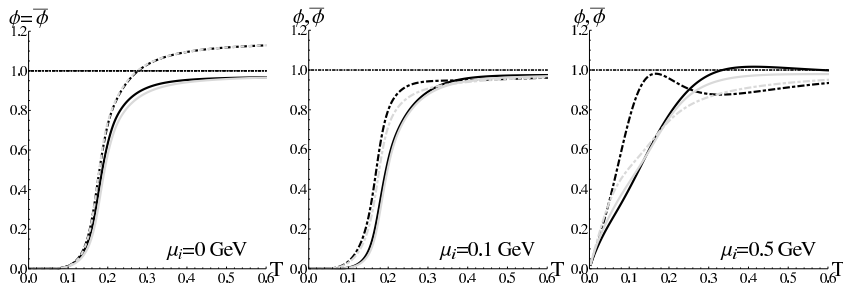
# Results using $U'$ in the PNJLH model



Incorrect behaviour for  $\nu$  only with 3D cutoff everywhere.

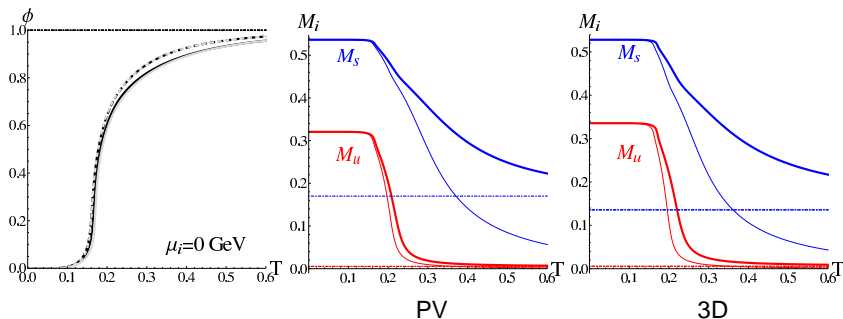
Whitout cutoff  $\lim_{T \rightarrow \infty} M = 0$ .

# Results using $\mathcal{U}^I$ in the PNJLH model



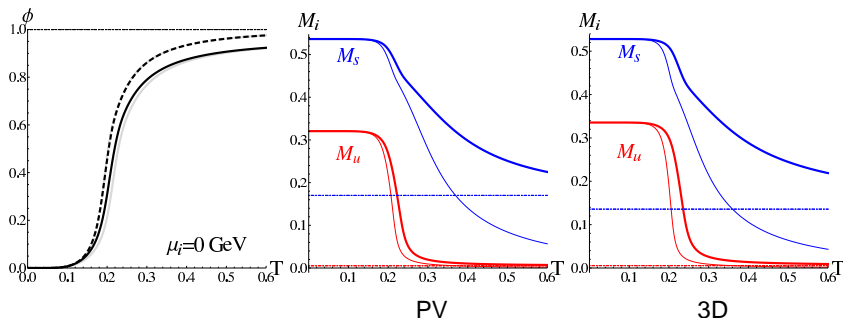
With cutoff everywhere we obtain  $\lim_{T \rightarrow \infty} \phi, \bar{\phi} = 1$

# Results using $\mathcal{U}^{\parallel}$ in the PNJLH model



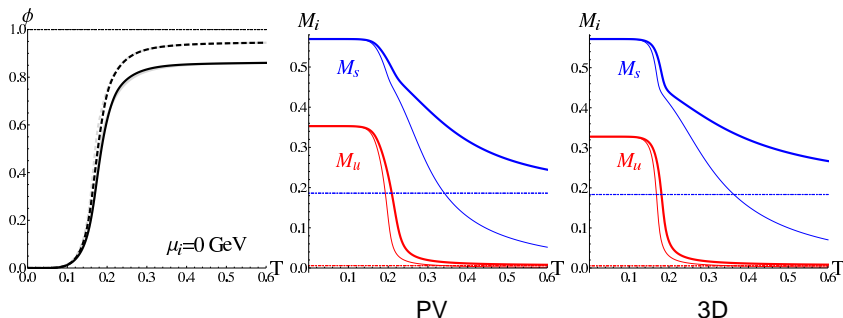
Whitout cutoff  $\lim_{T \rightarrow \infty} M = 0$ .

# Results using $u^{III}$ in the PNJLH model



Without cutoff  $\lim_{T \rightarrow \infty} M = 0$  and overshooting of the pure gluonic asymptotic solution without cutoff.

# Results using $U^IV$ in the PNJLH8q model



Without cutoff  $\lim_{T \rightarrow \infty} M = 0$  and overshooting of the pure gluonic asymptotic solution without cutoff.

# Some considerations on divergences with temperature

$$\frac{\partial}{\partial \phi} J_{-1}(M^2 = 0, \mu = 0, T, \phi, \phi)$$

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- $\lim_{T \rightarrow \infty} \frac{1}{T^4} \frac{\partial}{\partial\phi} J_{-1}^{3D, \infty} \propto f_2(\phi)$
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$$\Rightarrow \sum \frac{\partial J_{-1}}{\partial \phi} + A \frac{\partial \mathcal{U}}{\partial \phi} = 0$$

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- Overshooting of the Polyakov loop asymptotic solutions with  $\Lambda \rightarrow \infty$
- $\hat{\rho}^{PV}$  passes all these tests
- No conclusion as for the choice of Polyakov potential

# Main references

- B. Hiller, J. Moreira, A.A. Osipov, A.H. Blin, Phys. Rev. D 81, 116005 (2010); 0812.1532 [hep-ph]
- J. Moreira, B. Hiller, A.A. Osipov, A.H. Blin, 1008.0569 [hep-ph]