On Locating the Critical End Point in QCD Phase Diagram

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Reference : **QCD Critical Point in Quasiparticle Model ,**

P. K. Srivastava, S. K. Tiwari and C. P. Singh,

Phys. Rev. D82, 014023 (2010)

Outline :

- Conjectured QCD Phase Diagram and Critical End Point
- EOS for Hadron gas (HG)

  Excluded volume correction

  Results for Hadron yields & Freeze-out points

- EOS for Quark Gluon Plasma (QGP)

  Quasiparticle Model I (QPM I)

  Quasiparticle Model II (QPM II)

  Comparison : Quasiparticle Models Vs. Lattice QCD

- Deconfinement Phase Diagram and Critical End Point
- Conclusions & Inferences

Conjectured QCD Phase Diagram and Critical End point (CEP)

Features:

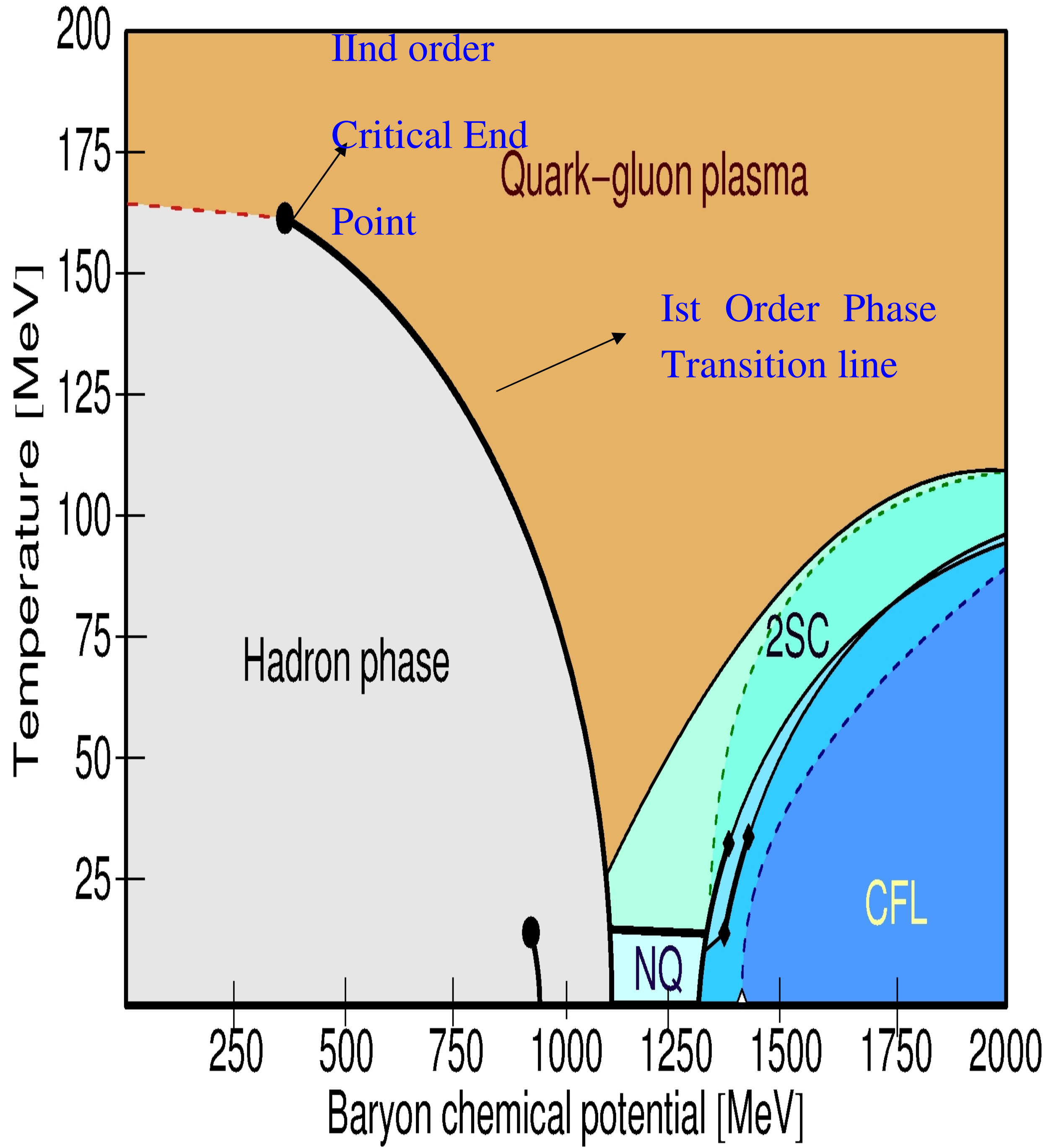
(1) Lattice QCD finds a rapid, but smooth crossover at large T and μ_B=0.

(2) Cross over region ends at a critical end point.

(3) Various effective chiral models find a strong 1st order transition beyond critical end point when T and μ_B are moderately large.

(4) Coordinates of CEP in (T,μ_B) plane vary wildly.

(5) Intuitive physical explanations for CEP and cross over do not exist.



Lattice Gauge Theories

Features :

- Lattice calculations unreliable for $\mu_B \neq 0$ region.

  Absence of probability measure & Sign problem

- Imaginary μ_B : de Forcrand & O. Phillipsen --> no CEP

- Taylor expansion of the pressure : Karsch; Gavai & Gupta

  ---> CEP

So, we need effective chiral models like NJL or other

phenomenological models.

A new Model for Hadron Gas EOS

Features :

- We have used **full quantum statistics** (in earlier version by Mishra et al., Boltzmann statistics has been used) to explore the entire T-μ_B plane.
- The excluded volume model is **thermodynamically consistent** as we start from the partition function and we get number density from it.
- We have used all the **hadrons and their resonances upto mass of 2 GeV** (to include the effect of attractive interactions) in the HG.
- We have considered a **hard-core size for each baryon** to include the effect of repulsive interactions at high density.
- **Mesons are treated as a pointlike particles** because they don't have any hard - core size and they overlap and fuse with each other.
- We **obtain hadron yields** and compare with the experimental data. This gives **confidence in EOS** for HG.
- We calculate the **chemical freeze out curve** on the phase boundary and show the **proximity of CEP to this curve.**

EOS for the HG

The Grand canonical partition function -:

$$\ln Z_i^{ex} = \frac{g_i}{6\pi^2 T} \int_{V_i^0}^{V-\sum_j N_j V_j} dV \int_0^\infty \frac{k^4 dk}{\sqrt{k^2+m_i^2}} \frac{1}{[\exp(\frac{E_i-\mu_i}{T})+1]}$$

Where g_i is the degeneracy factor of ith species of baryon, E is the energy of the particle V^0_i is the eigen volume of one ith species of baryon and $\sum_j N_j V_j^{\ 0}$ is the total volume occupied by baryons.

We can rewrite above equation as $\ln Z_i^{ex} = V(1-\sum_j n_j^{ex} V_j^0) I_i \lambda_i$

Where $$I_i = \frac{g_i}{6\pi^2 T} \int_0^\infty \frac{k^2 dk}{\sqrt{k^2+m_i^2}} \frac{1}{[\exp(\frac{E_i}{T})+\lambda_i]}$$

$\lambda_i = \exp(\frac{\mu_i}{T})$ is the fugacity of the particle, n^{ex}_j is the number density of jth type of baryons after excluded volume correction.

Using the basic thermodynamical relation between number density and partition function

We can write as -

$$n_i^{ex} = (1-R)I_i\lambda_i - I_i\lambda_i^2 \frac{\partial R}{\partial \lambda_i} + \lambda_i^2(1-R)I_i^{'}$$

Where $R = \sum_i n_i^{ex} V_i^0$ is the fractional volume occupied. We can write R in an operator equation - $R = R + \hat{\Omega} R$ Where $R = \frac{R^0}{1+R^0}$ with $R_i^0 = \sum n_i^0 V_i^0 + \sum I_i^{'} V_i^0 \lambda_i^2$

And the operator $\hat{\Omega} = -\frac{1}{1+R^0}\sum_i n_i^0 V_i^0 \lambda_i \frac{\partial}{\partial \lambda_i}$

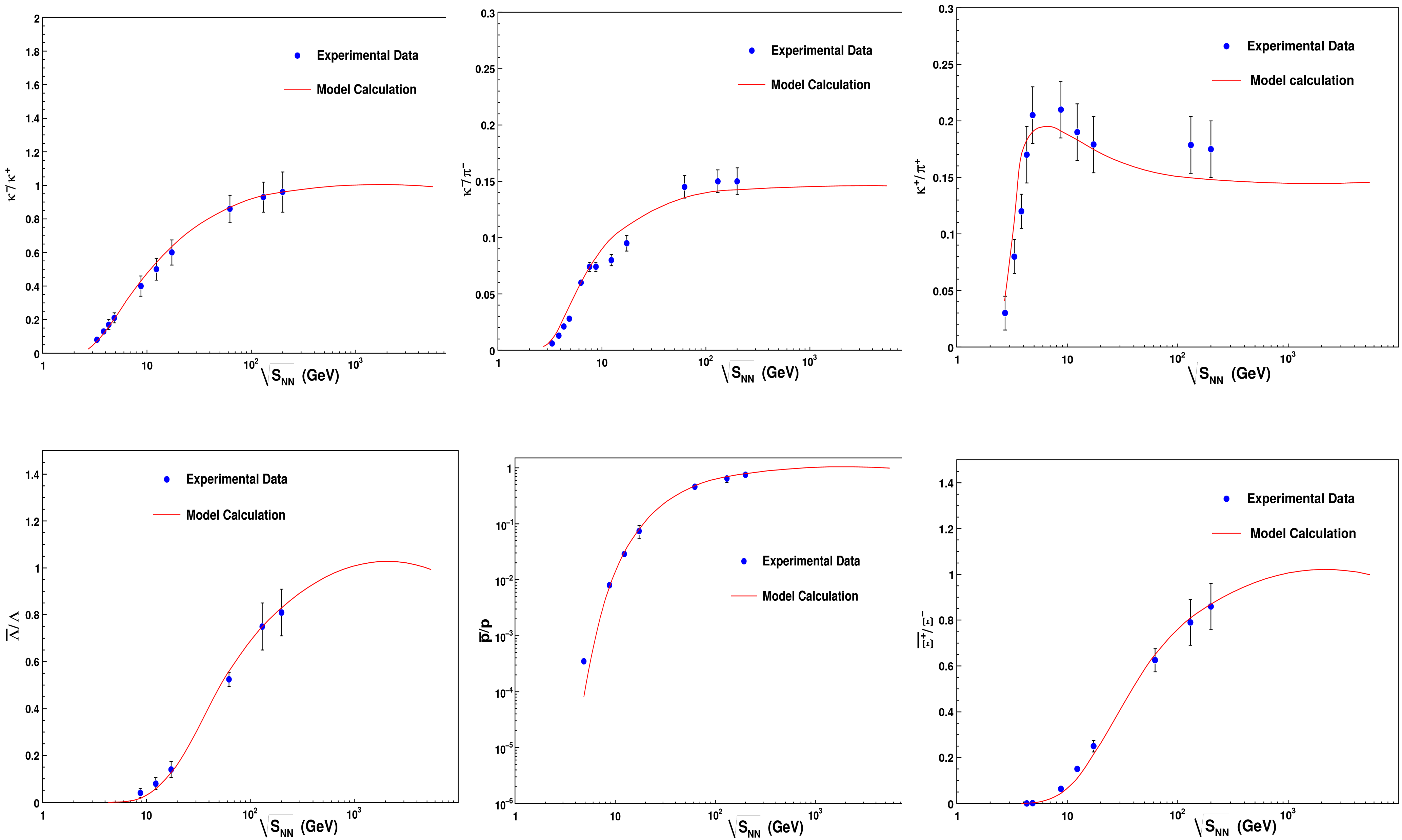
Using Neumann iteration method, we get after truncation upto $\hat{\Omega}^2$ term

(further terms give small contribution) :

$$R = R + \hat{\Omega} R + \hat{\Omega}^2 R$$

Solving this equation numerically, we can get R and hence the total pressure of hadron gas after Excluded volume correction is - $P_{HG}^{ex} = T(1-R)\sum_i I_i\lambda_i + \sum_i P_i^{meson}$

Particle ratios calculated by our model and its comparison with data from Heavy Ion collision experiments. [Ref: C. Blume, J. Phys. G 31, S57 (2005); A. Andronic et. al., arXiv:0911.4806v3 [hep-ph]



EOS for QGP---> Quasiparticle Model

- Quasiparticles are the thermal excitations of the interacting quarks and gluons.
- First proposed by Goloviznin and Satz and then by Peshier et al. [Ref: V. Golovinznin and H. Satz, Z. Phys. C 57, 671 (1993); A. Peshier et. al., Phys. Rev. D 54, 2399 (1996)]
- The Quasiparticle Quark Gluon Plasma(qQGP) is a phenomenological model, used to describe the non ideal behaviour of quark gluon plasma (QGP)

Quasiparticle Model

System of interacting massless particles $\xrightarrow{\text{effectively}}$ Ideal gas of "massive" non-interacting particles

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[Ref: A. Peshier et. al., Phys. Rev. D 54, 2399 (1996)]

Dispersion relation for these massive particles is assumed as :

Energy $$\omega^2(k,m) = k^2 + m^2(T)$$

Where m(T) $\rightarrow$ temperature - dependent mass

and k $\longrightarrow$ momentum of the particle

Effective masses : (Using Finite temperature field theory)

For Gluons $$m_g^2(T) = \frac{N_C}{6} g^2(T) T^2 \left(1 + \frac{N_f^{'}}{6}\right)$$ Where N_C is the no. of colours

$$N_f^{'} = N_f + \frac{3}{\pi^2} \sum_f \frac{\mu_f}{T^2}$$ Where N_f is no. of flavours of quarks and μ_f is chemical potential

For quarks $$m_{qf}^2(T) = \frac{g^2(T) T^2}{6} \left(1 + \frac{\mu_f^2}{\pi^2 T^2}\right)$$

g^2 (T) is the QCD running coupling constant [Ref : V. M. Bannur, Eur. Phys. J. C 50, 629 (2007)]

$$g^2(T,\mu) = \frac{24\pi^2}{\left(33-2n_f\right)\ln\left(\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu^2}{T^2}}\right)}\left(1-\frac{3(153-19n_f)\ln(2\ln\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu^2}{T^2}})}{(33-2n_f)^2\ln\left(\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu^2}{T^2}}\right)}\right)$$

Pressure of QGP $\longrightarrow$ $$P_{id} = \mp\frac{Td}{2\pi^2}\int_0^\infty k^2 dk \ln\left[1 \mp \exp\left(-\frac{\left(\omega-\mu_q\right)}{T}\right)\right]$$

Energy density of QGP $\longrightarrow$ $$\varepsilon_{id} = \frac{d}{2\pi^2}\int_0^\infty \frac{k^2 dk\omega}{\exp\left\{\left(\omega-\mu_q\right)/T\right\} \mp 1}$$

But this model doesn't satisfy the thermodynamic relation $\longrightarrow$

$$\varepsilon\left(T\right) = T\frac{dP(T)}{dT} - P(T)$$

Thermodynamical Inconsistency

Two different approaches were proposed to remove this " inconsistency"

(1) Quasiparticle Model I (QPM I)

[Ref:A. Peshier et. al., Phys. Rev. D 54, 2399 (1996)]; M. I. Gorenstein and S. N. Yang, Phys. Rev. D52, 5206 (1995)]

In this model the above inconsistency is handled by introducing a "temperature dependent vacuum energy" term which effectively cancelled the inconsistent term.

So the pressure and energy density for the system of quasiparticles can be written in a thermodynamically consistent manner as follows :

$$P = P_{id} - B(T,\mu)$$

$$\varepsilon = \varepsilon_{id} + B(T,\mu)$$

Where $B(T,\mu) = B_0 - \frac{d}{4\pi^2}\int_{T_0}^{T} dT \frac{dm^2(T)}{dT}\int_0^{\infty} \frac{k^2 dk}{\omega\{\exp((\omega-\mu_q)/T) \mp 1\}}$

In our calculation we taken $B_0^{1/4}$=185 MeV and T_0=100 MeV.

(2) Quasiparticle Model II (QPM II) [Ref : V. M. Bannur, Eur. Phys. J. C 50, 629 (2007); Phys. Lett. B647, 271 (2007)]

In QPM II , Bannur has suggested that the relation between pressure and grand canonical partition function cannot hold good if the particles of the system have a temperature-dependent mass. So we should start with the definition of average energy density and average number of particles and derived all the thermodynamical quantities from them in a consistent manner.

$$\varepsilon = \frac{T^4}{\pi^2} \sum_{l=1}^{\infty} \frac{1}{l^4} \left[\frac{d_g}{2} \varepsilon_g (x_g l) + (-1)^{l-1} d_q \cosh\left(\frac{\mu_q}{T} \right) \varepsilon_q (x_q l) + (-1)^{l-1} \frac{d_s}{2} \varepsilon_s (x_s l) \right]$$

$$n_q = \frac{d_q T^3}{\pi^2} \sum_{l=1}^{\infty} (-1)^{l-1} \frac{1}{l^3} \sinh(\mu_q / T) I_i (x_i l)$$

with $\varepsilon_i (x_i l) = (x_i l)^3 K_1 (x_i l) + 3 (x_i l)^2 K_2 (x_i l)$

Where K_1 and K_2 are the modified bessel's functions.

In this model , the effective mass of the quarks is :

$$m_q^2 (T) = m_{q0}^2 + \sqrt{2} m_{q0} m_{th} + m_{th}^2$$

Where m_{q0} is the rest mass of the quarks. In our calculation, we have used m_{q0}=8 MeV for two light quarks (u ,d), and m_{q0}=80 MeV for strange quarks. In the above Eq. m_{th} represents the thermal mass of the quarks.

$$m_{th}^2(T,\mu) = \frac{N_C^2 - 1}{8N_C}\left[T^2 + \frac{\mu_q^2}{\pi^2}\right]g^2$$

Pressure of the system at μ_q =0 can be obtained as :

$$\frac{P(T,0)}{T} = \frac{P_0}{T_0} + \int_{T_0}^{T} dT \, \frac{\varepsilon(T,\mu=0)}{T^2}$$

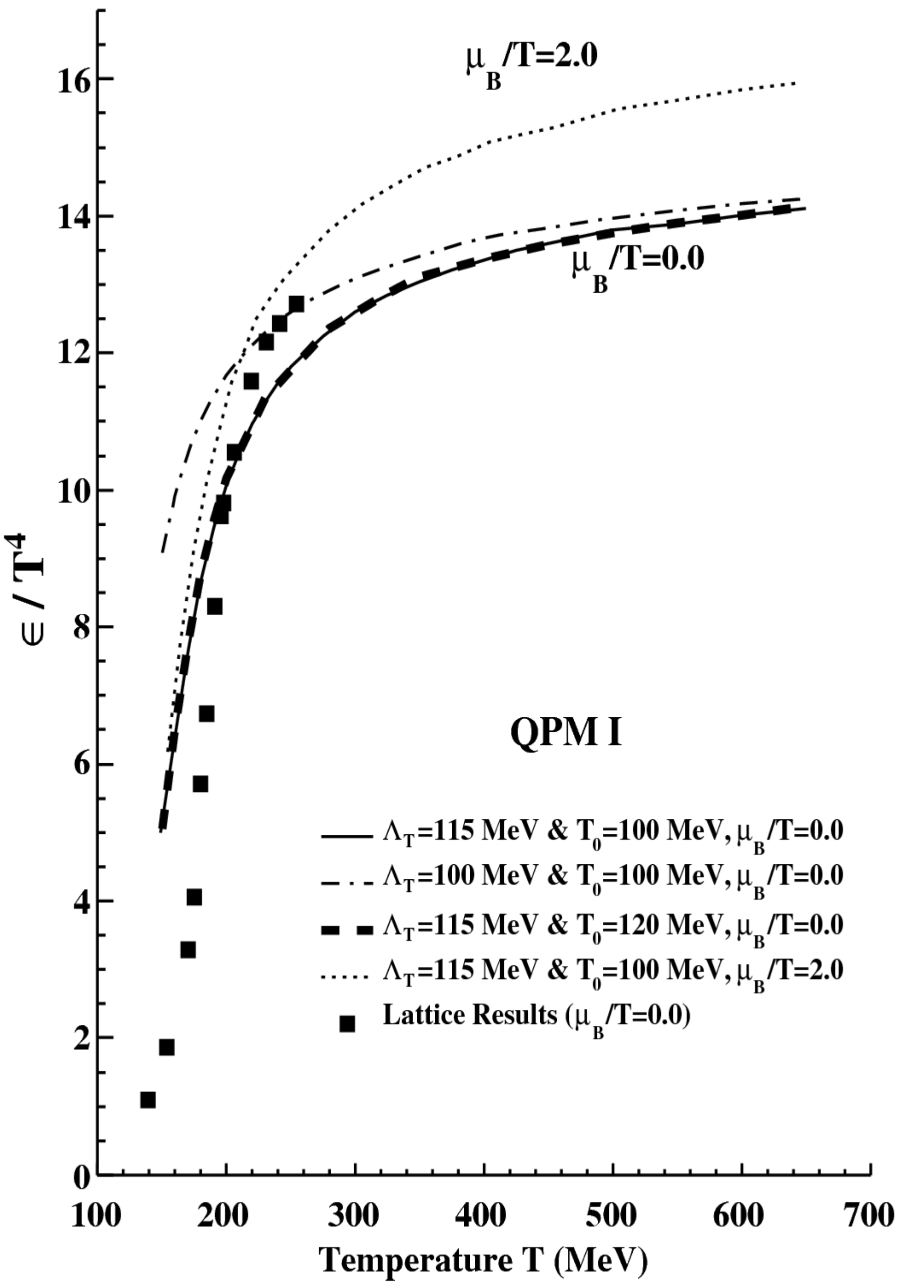
Where P_0 is the pressure at a reference temperature T_0 . We have used $P_0 = 0$ at T_0=100 MeV in our calculation.

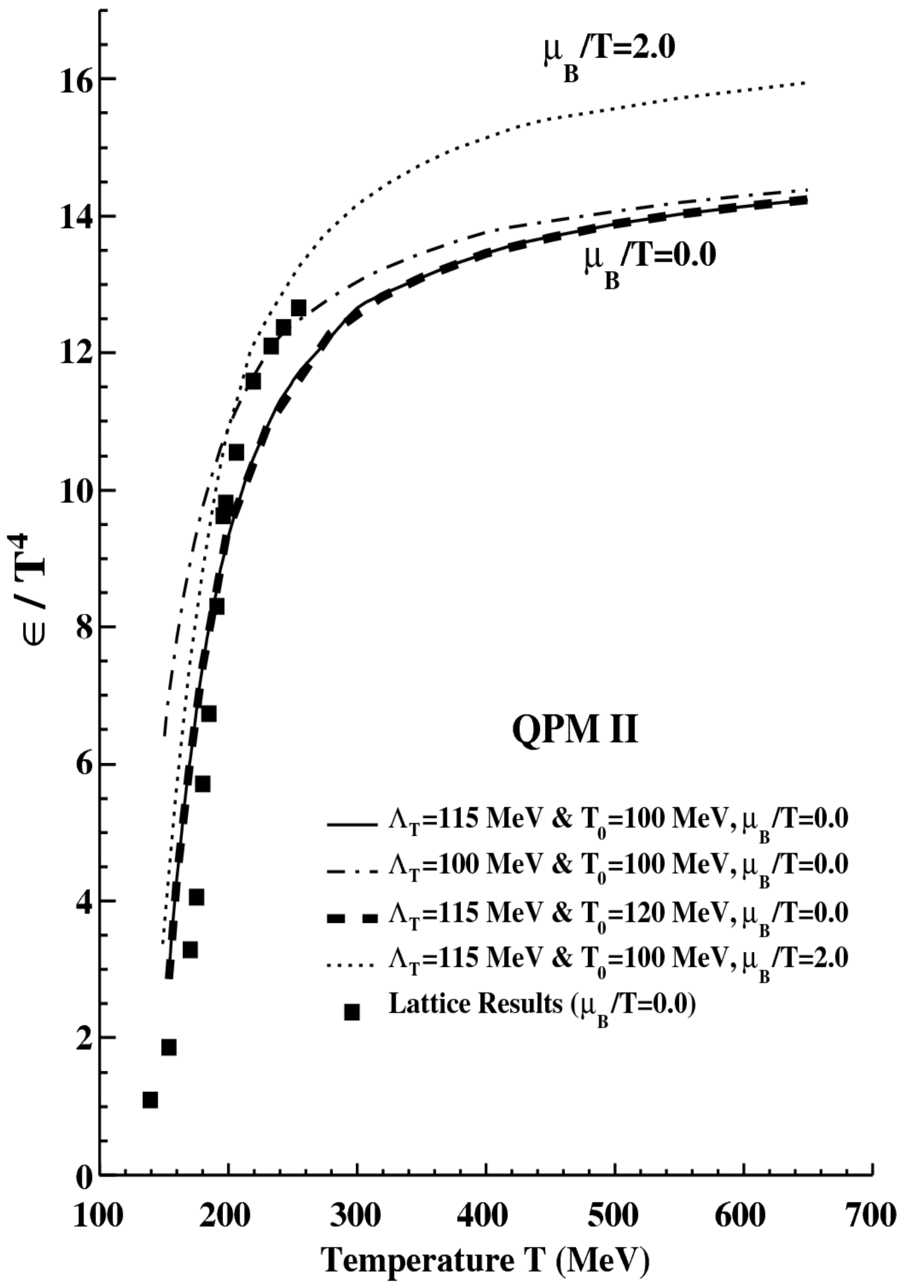
Using the relation between the number density and the grand canonical partition function, we can get the pressure for a system at finite μ_q.

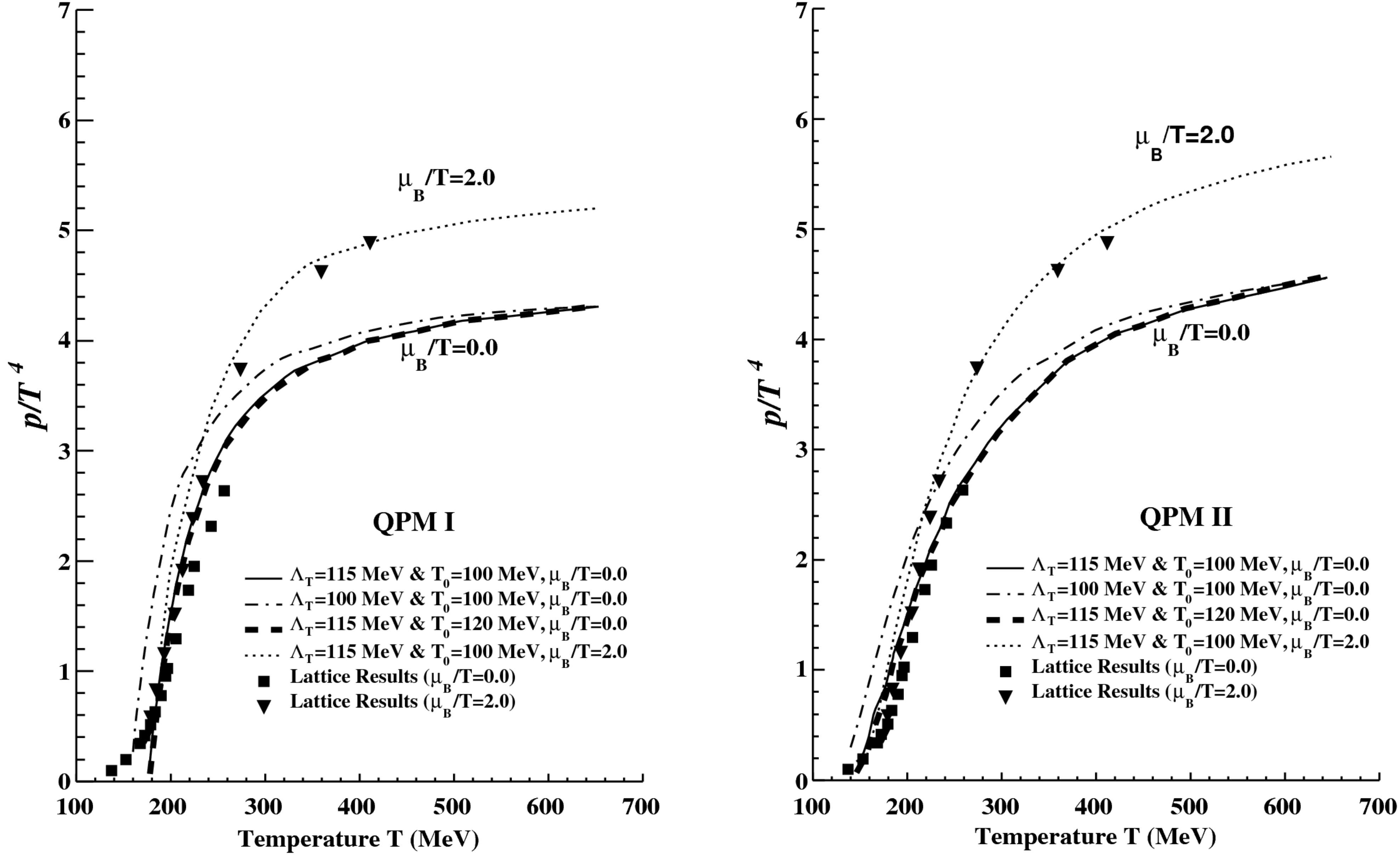
$$P(T,\mu_q) = P(T,0) + \int_0^{\mu_q} n_q \, d\mu_q$$

Comparison : Quasiparticle Models Vs. Lattice data

To fix the values of the parameters







We have chosen the solid line as best fit and taken the values of the parameters from this fit. (Λ_T=115 MeV and T_0=100 MeV)

Deconfinement transition using Gibbs' Criteria :

- Using Maxwell construction, first order phase transition occurs when: $P_H(T_c,\mu_C) = P_Q(T_c,\mu_C)$, this ensures mechanical, chemical and thermal equilibrium between HG and QGP phases.
- In cross over region, $P_Q > P_H$ (always), so mostly pions and quarks, gluons coexist.
- End point of first order line is taken as critical end point.
- We get deconfining first order phase transition curve. We compare with the phase boundary obtained in the earlier analysis where we used Bag model EOS for QGP. Reference : QCD Phase Boundary and Critical Point in a Bag Model Calculation C. P. Singh, P. K. Srivastava, S. K. Tiwari, Phys. Rev. D 80, 114508 (2009)
- We have used two different QGP EOS based on quasiparticle models which explain the lattice data very well.

QCD Phase diagram using Quasiparticle Model EOS for QGP

P_1 --> Ist order phase transition curve in Bag Model

BM--> End point in Bag Model

P_2--> Ist order phase transition curve in QPM I

C_1--> End point in QPM I

P_3--> Ist order phase transition curve in QPM II

C_2--> End Point in QPM II

F --> Chemical Freeze-out curve

All other labels from--> **P.K.S. , S.K.T. & C.P.S.,**

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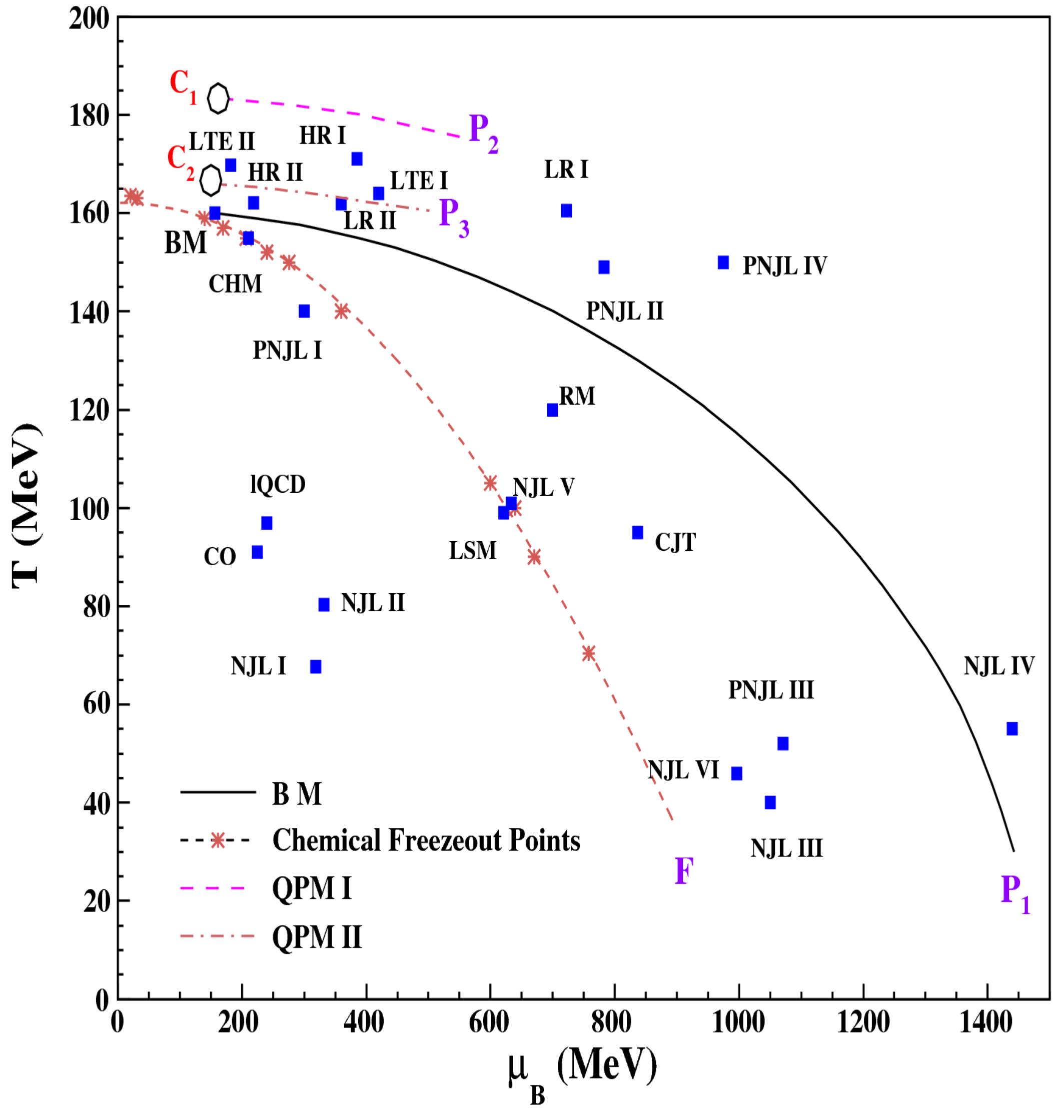

Conclusions & Inferences:

- *We have given a new thermodynamically consistent excluded volume model which fits well the hadron yields data of RHIC. We have determined the chemical freeze out curve and compare its proximity to CEP.*

- *We have used two different versions of QGP EOS based on quasiparticles which reproduce the lattice data very well.*

- *By Maxwell construction, we got a first order deconfining phase transition : $P_H (T_c, \mu_C) = P_Q (T_c, \mu_C)$ which ends on a critical point. Usually other models determine CEP in chiral phase transition.*

- *We have got a crossover region beyond critical end point where $P_Q > P_H$ (always) and so we infer that quarks, gluons and mesons coexist in this region.*

- *The critical end point found in this model supports our previous result (PRD 80, 114508 (2009)) obtained in Bag model.*