Chiral symmetry breaking in strong magnetic background

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Plan of talk

1. Introduction
   - Chiral symmetry breaking in vacuum.
   - Chiral symmetry breaking in magnetic background.
2. Solution of Dirac equation in magnetic field.
3. Ansatz for the ground state and evaluation of the thermodynamic potential.
4. Mass gap equation.
5. Results and discussions.

Introduction

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   - Relevant for heavy ion collision experiments (Mag. field $\sim 10^{18}$ Gauss).

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    - Strong magnetic field exists in compact stars.
    - Relevant for heavy ion collision experiments (Mag. field $\sim 10^{18}$ Gauss).
4. NJL model for studying χSB.

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5. For $E < 0$, the energy levels are given by

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Where

$$B^\dagger = \sum_n \int d\mathbf{p}_{\not{x}}\, q_r^\dagger(n, \mathbf{p}_{\not{x}}) a_{r,s}(n, p_z) f(n, \mathbf{p}_{\not{x}}) \tilde{q}_s(n, -\mathbf{p}_{\not{x}})$$

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3. $f(n, \mathbf{p}_{\not x})$ is the condensate function. and

$$a_{r,s} = \frac{1}{\sqrt{p_z^2 + 2n|q_i|B}} \left[-\sqrt{2n|q_i|B}\,\delta_{r,s} - ip_z \delta_{r,-s} \right]$$

Chiral condensate

1. The chiral condensate term is given by

$$\langle\bar{\psi}\psi\rangle_i = -\sum_{n=0}^{\infty}\frac{N_c|q_i|B\alpha_n}{(2\pi)^2}\int dp_z\frac{\cos 2\theta^i_{\pm}}{\epsilon_{ni}}\left[m_i\cos 2f_i + |p_i|\sin 2f_i\right]$$
$$= -I_i$$

where

$$\epsilon_{ni} = \sqrt{m_i^2 + p_z^2 + 2n|q_i|B}$$

and

$$\cos 2\theta^i_{\pm} = 1 + \sin^2\theta^i_{-} + \sin^2\theta^i_{+}$$

3 − *flavor* NJL Lagrangian

1. The 3 − *flavor* NJL Lagrangian in presence of magnetic field with the Kobayashi-Maskawa-t'Hooft [KMT] vertex is given by

$$\mathcal{L} = \bar{\psi}\,(i\partial\!\!\!/ - q_i B x \alpha_2 - m)\,\psi + G\sum_{A=0}^{8}\left[(\bar{\psi}\lambda^A\psi)^2 + (\bar{\psi}\gamma^5\lambda^A\psi)^2\right]$$
$$- K\left[det\{\bar{\psi}(1+\gamma^5)\psi\} + det\{\bar{\psi}(1+\gamma^5)\psi\}\right]$$

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2. λ's are the Gell Mann matrices satisfying

$$\sum_{A=0}^{8}\lambda_{ij}^A\lambda_{kl}^A = 2\delta_{il}\delta_{jk}$$

Thermodynamic potential

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2. S is the entropy which is given by

$$\begin{aligned} S &= -\sum_i \sum_n \frac{N_c \alpha_n |q_i| B}{(2\pi)^2} \int dp_z \{\sin^2\theta^i_- \ln \sin^2\theta^i_- + - \leftrightarrow +\} \\ &+ \int dp_z \{\cos^2\theta^i_- \ln \cos^2\theta^i_- + - \leftrightarrow +\} \end{aligned}$$

$T = 0$, $\mu = 0$ case

1. The thermodynamic potential is given by

$$\Omega = -\sum_{n=0}^{\infty}\sum_{i}\frac{N_c\alpha_n|q_iB|}{(2\pi)^2}\int dp_z\sqrt{M_i^2+|p_i|^2}+2G\sum_i I_i^2+4KI_1I_2I_3$$

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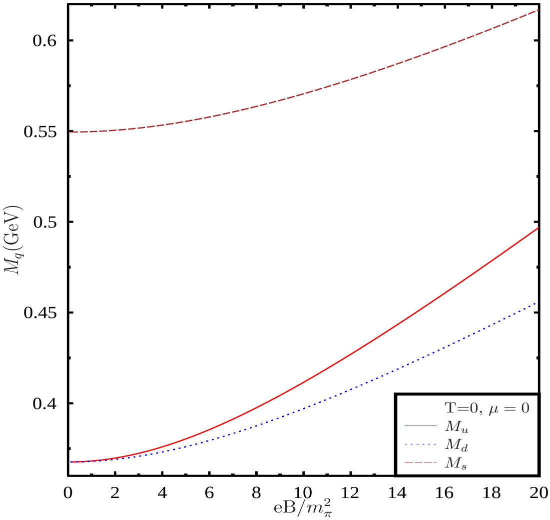
2. The gap equation at $T = 0$, $\mu = 0$ is given by

$$M_i = m_i + 4GI_i + 2K|\epsilon_{ijk}|I_jI_k$$

$T = 0,\ \mu = 0$ case



M_q(GeV)
0.6
0.55
0.5
0.45
0.4
0 2 4 6 8 10 12 14 16 18 20
eB/m_π^2
T=0, $\mu = 0$
M_u
M_d
M_s

$T \neq 0$, $\mu \neq 0$ case

1. The thermodynamic potential is given by

$$\begin{aligned}\Omega &= -\frac{2N_c}{(2\pi)^3}\sum_i \int d^3p\sqrt{\mathbf{p}_i^2 + M_i^2} + 2G\sum_i I_i^2 + 4KI_1I_2I_3 \\ &- \frac{N_c}{2\pi^2}\sum_i |q_iB|^2\left[\zeta'(-1,x_i) - \frac{1}{2}(x_i^2 - x_i)\ln x_i + \frac{x_i^2}{4}\right] \\ &- \sum_{n,i}\frac{N_c\alpha_n|q_iB|}{(2\pi)^3\beta}\int dp_z \ln\left\{1 + e^{-\beta(\omega_i - \mu_i)}\right\} \\ &- \sum_{n,i}\frac{N_c\alpha_n|q_iB|}{(2\pi)^3\beta}\int dp_z \ln\left\{1 + e^{-\beta(\omega_i + \mu_i)}\right\}\end{aligned}$$

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Where $x_i = \frac{M_i^2}{2|q_i B|}$. and

$$\omega_{ni} = \sqrt{M_i^2 + |p_i|^2}$$

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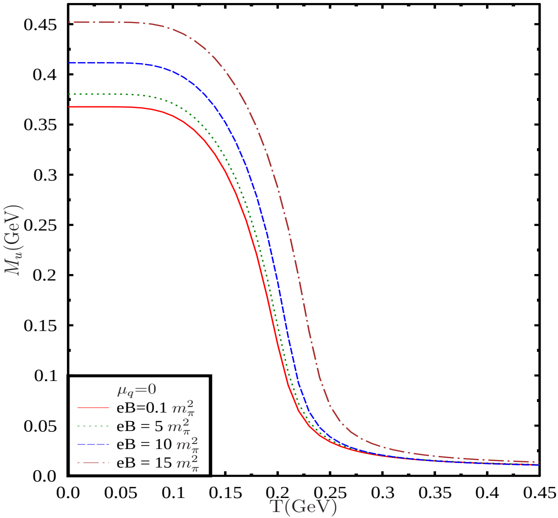
2. The chiral condensate term is given by

$$\begin{aligned}
I_i &= I^i_{vac} + I^i_{field} + I^i_{med} \\
&= \frac{N_c M_i}{2\pi^2}\left[\Lambda\sqrt{\Lambda^2 + M_i^2} - m_i^2 \ln\left(\frac{\Lambda + \sqrt{\Lambda^2 + M_i^2}}{M_i}\right)\right] \\
&+ \frac{N_c M_i |q_i B|}{(2\pi)^2}\left[x_i(1 - \ln x_i) + \ln\Gamma(x_i) + \frac{1}{2}\ln\frac{x_i}{2\pi}\right] \\
&- \sum_{n=0}^{\infty}\frac{N_c|q_i|B\alpha_n}{(2\pi)^2}\int dp_z \frac{M_i}{\sqrt{M_i^2 + |p_i|^2}}(\sin^2\theta^i_- + \sin^2\theta^i_+)
\end{aligned}$$

where Λ is the cut off in NJL model.

$T \neq 0$, $\mu = 0$ case



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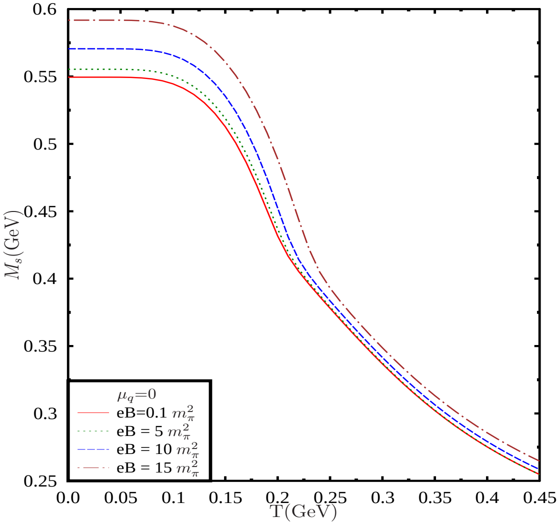

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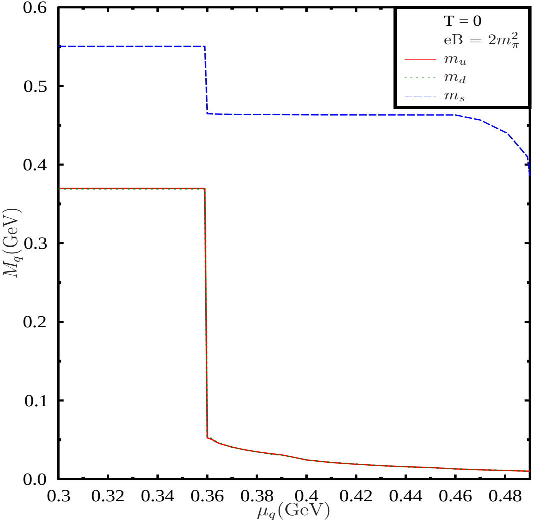

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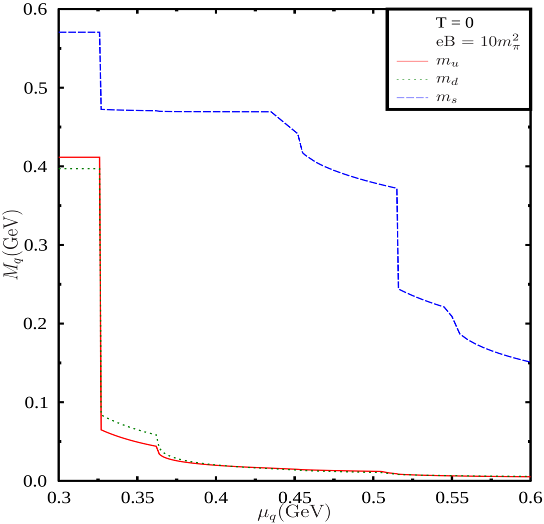
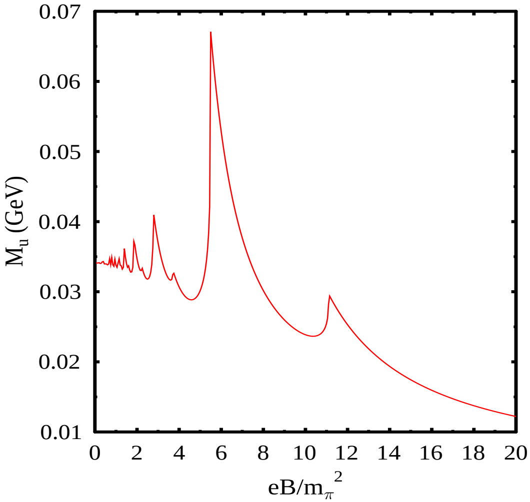


de Haas-van Alphen effect

1. At $T = 0$ gaps depend on the density of states of quarks at the Fermi surface.
2. Density of state has an oscillatory structure because of different quantization levels with varying magnetic field.
3. Gaps display magnetic oscillation.
4. Similar to de Haas-van Alphen effect in metals.

1. de Haas-van Alphen effect. We have taken $\mu = 380$ MeV.

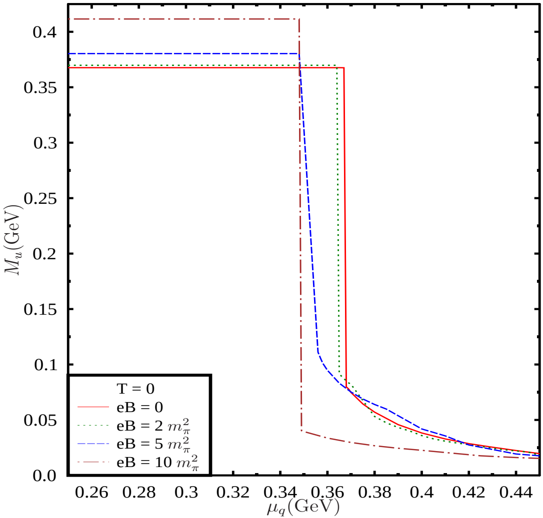


1. Charge neutrality demands

$$\mu_s = \mu_d = \mu_u + \mu_e.$$

2. We need 2 independent chemical potentials. $\mu_q = \frac{1}{3}\mu_B$ and μ_e.

Charge neutrality at T = 0



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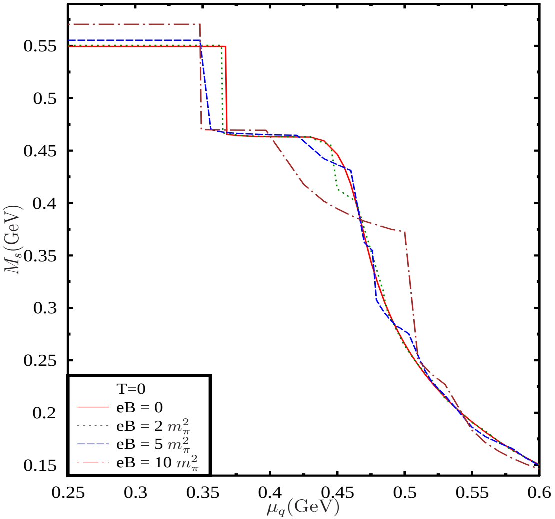
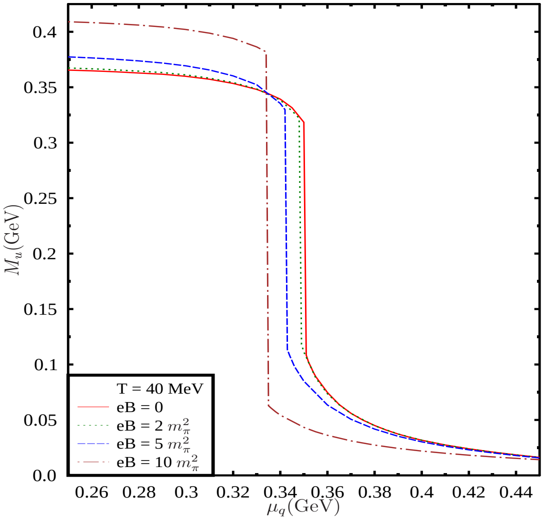


Charge neutrality at T $\neq$ 0



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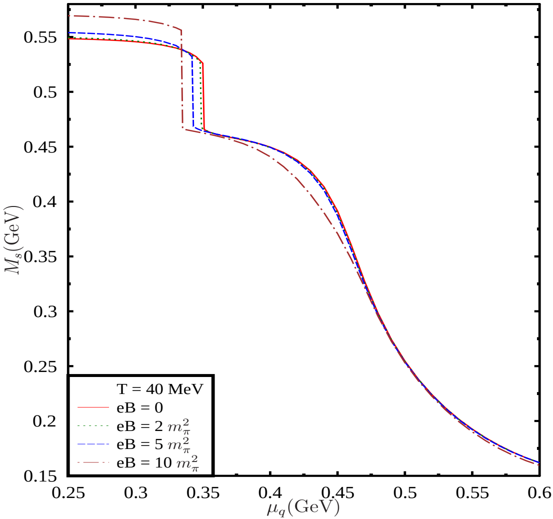
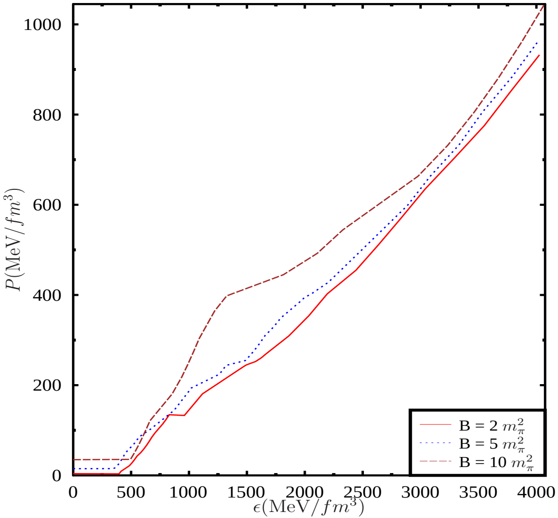


Equation of state at T = 0



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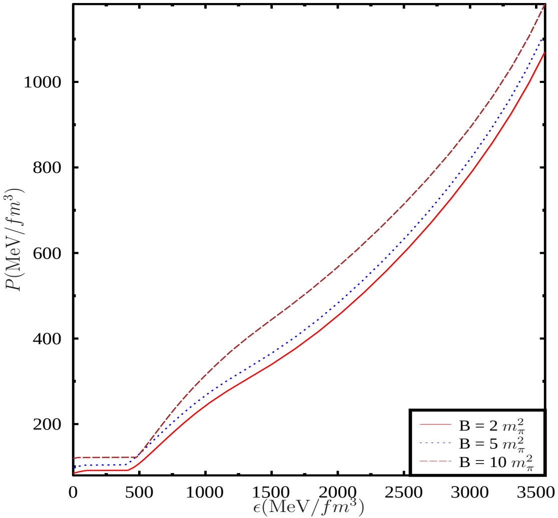

Collaborators

1. Dr. Hiranmaya Mishra [PRL, Ahmedbad]
2. Dr. Amruta Mishra [IIT Delhi]

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Thank You

Dirac spinors in magnetic field

1. The Dirac spinors for particles with positive electrical charge are given by

$$U_{\uparrow}(\mathbf{p}_{\not{x}}, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} (\epsilon_n + m) I_n \\ 0 \\ p_z I_n \\ -i\sqrt{2nq_i B} I_{n-1} \end{bmatrix} e^{i\mathbf{p}_{\not{x}} \cdot \mathbf{x}_{\not{x}}} e^{-i\epsilon_n t}$$

and

$$U_{\downarrow}(\mathbf{p}_{\not{x}}, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} 0 \\ (\epsilon_n + m) I_{n-1} \\ i\sqrt{2nq_i B} I_n \\ -p_z I_{n-1} \end{bmatrix} e^{i\mathbf{p}_{\not{x}} \cdot \mathbf{x}_{\not{x}}} e^{-i\epsilon_n t}$$

Dirac spinors in magnetic field

1. The Dirac spinors for anti particles with positive electrical charge are given by

$$V_{\uparrow}(-\mathbf{p}_{\not{x}}, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} \sqrt{2nq_iB}I_n \\ ip_z I_{n-1} \\ 0 \\ (i\epsilon_n + m)I_{n-1} \end{bmatrix} e^{-i\mathbf{p}_{\not{x}}\cdot\mathbf{x}_{\not{x}}} e^{i\epsilon_n t}$$

and

$$V_{\downarrow}(-\mathbf{p}_{\not{x}}, n) = \frac{1}{\sqrt{2\epsilon_n(\epsilon_n + m)}} \begin{bmatrix} ip_z I_n \\ \sqrt{2nq_iB}I_{n-1} \\ -i\epsilon_n + m)I_n \\ 0 \end{bmatrix} e^{-i\mathbf{p}_{\not{x}}\cdot\mathbf{x}_{\not{x}}} e^{i\epsilon_n t}$$

Dirac spinors in magnetic field

1. I_n is the solution of Hermite differential equation given by

$$I_n(\xi) = C_n e^{-\frac{\xi^2}{2}} H_n(\xi)$$

Where $H_n(\xi)$ is the Hermite polynomial and $C_n = \left[\frac{\sqrt{q_i B}}{n! 2^n \sqrt{\pi}}\right]^{\frac{1}{2}}$.
$\xi = \sqrt{q_i B}\left(x - \frac{p_y}{q_i B}\right)$ is a dimensionless variable.

2. I_n's follow the following orthonormality relation

$$\int d\xi I_n(\xi) I_m(\xi) = \sqrt{|q_i| B}\delta_{n,m}$$

Important expectation values

1. The free particle energy term is given by

$$
\begin{aligned}
T + m\langle\bar{\psi}\psi\rangle &= \langle\psi^\dagger(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} - q_i Bx\alpha_2 + \beta m)\psi\rangle \\
&= -\sum_{n=0}^{\infty}\sum_i \frac{N_c\alpha_n|q_i B|}{(2\pi)^2}\int dp_z\epsilon_{ni}\cos 2\theta^i_\pm \cos 2f_i
\end{aligned}
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2. The number density term is given by

$$\langle\psi^\dagger\psi\rangle = \sum_{n=0}^{\infty}\sum_i \frac{N_c \alpha_n |q_i B|}{(2\pi)^2}\int dp_z \left[1 - \sin^2\theta^i_+ + \sin^2\theta^i_-\right]$$

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2. The kinetic energy is given by

$$T = -\sum_{n=0}^{\infty} \sum_{i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z \epsilon_{ni} \cos 2f_i$$

with

$$\epsilon_{ni} = \sqrt{m_i^2 + p_z^2 + 2n|q_i|B} = \sqrt{m_i^2 + |p_i|^2}$$

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3. The interaction term is given by

$$\begin{aligned} V &= -\langle G\sum_{A=0}^{8}\left[(\bar{\psi}\lambda^A\psi)^2 + (\bar{\psi}\gamma^5\lambda^A\psi)^2\right]\rangle \\ &+ K\langle\left[det\{\bar{\psi}(1+\gamma^5)\psi\} + det\{\bar{\psi}(1+\gamma^5)\psi\}\right]\rangle \\ &= V_s + V_d \end{aligned}$$

$T = 0,\ \mu = 0$ case

1. In our case

$$\langle \bar{\psi}\gamma^5\psi \rangle = 0$$

and

$$det(\bar{\psi}_i\psi_j) = det\begin{bmatrix} \bar{\psi}_1\psi_1 & \bar{\psi}_2\psi_1 & \bar{\psi}_3\psi_1 \\ \bar{\psi}_1\psi_2 & \bar{\psi}_2\psi_2 & \bar{\psi}_3\psi_2 \\ \bar{\psi}_1\psi_3 & \bar{\psi}_2\psi_3 & \bar{\psi}_3\psi_3 \end{bmatrix}$$

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Therefore

$$V_d = 2K\langle\bar{\psi}_1\psi_1\rangle\langle\bar{\psi}_2\psi_2\rangle\langle\bar{\psi}_3\ \psi_3\rangle = -2KI_1I_2I_3$$

All other terms will be atleast $\frac{1}{N_c}$ suppressed.

2. We define $\frac{m_i}{\epsilon_{ni}} = \cos\phi_i^0$ and define $\phi_i = \phi_i^0 - 2f_i$

$T \neq 0,\ \mu \neq 0$ case

1. The thermodynamic potential is given by

$$\begin{aligned}\Omega &= -\sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^2} \int dp_z \omega_{ni} + 2G \sum_i I_i^2 + 4K I_1 I_2 I_3 \\ &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln\left\{1 + e^{-\beta(\omega_i - \mu_i)}\right\} \\ &- \sum_{n,i} \frac{N_c \alpha_n |q_i B|}{(2\pi)^3 \beta} \int dp_z \ln\left\{1 + e^{-\beta(\omega_i + \mu_i)}\right\}\end{aligned}$$

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Where

$$\omega_{ni} = \epsilon_{ni} = \sqrt{M_i^2 + |p_i|^2}$$

2. The first term is divergent.
3. For regularization, add and subtract a vacuum term

$$T_{vac} = \frac{2N_c}{(2\pi)^3} \sum_i \int d^3p \sqrt{\mathbf{p}_i^2 + M_i^2}$$